SCHOOL SCIENCE AND MATHEMATICS

VOL. VIII. No. 3 CHICAGO, MARCH, 1908 WHOLE No. 59

LECTURE EXPERIMENTS FOR TEACHING ELECTRICAL MEASUREMENTS FOR DIRECT AND ALTERNATING CURRENTS.

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I would have the teaching of electrical quantities run through all the teaching of electricity, even as symbols, formulas and equations run through all our teaching of chemistry. For this purpose I have installed at my lecture table volt-meters and ammeters with dials large enough to be read from any seat in the lecture room, and whenever the electric current is used for any experiment it is passed through these meters and attention is called to the quantities involved. Thus any experiment in decomposing a compound, lighting a lamp, ringing a bell, running a motor, etc., is made to teach incidentally electric measurements. The accompanying diagram will show my arrangement of the instruments and will be fully explained as we go on to describe some of the experiments.

V is an alternating current voltmeter and A is an alternating current ammeter, V′ and A′ are direct current meters and V″ and A″ are for either direct or alternating currents. The voltmeter scales run from 0 to 1.5 divided into hundredths, and multipliers, at *m*, make the instruments measure to 15, 30 and 300 volts. In the case of the alternating current voltmeter V, one multiplier carries the measurement to 1,200 volts. The ammeter scales run from 0 to 1 divided into hundredths, and shunts, at *sh*, make it possible to measure from one thousandth of an ampere to ten amperes. The 110 volt alternating current mains are represented at A. C. The 110 volt direct current mains are represented at D. C., S S and S′ S′ are double throw knife switches, S, S′ and S″ are single throw knife switches. B′ and B″ are pairs of binding posts for the attachments of apparatus, batteries, and other generators, and also the primary and secondary wires of transformers. R is a rheostat for controlling the current.

Experiment 1. Open the switch S, close S,′ and throw S′ S′ to the right so as to connect in the direct current meters V′ and A.′ Connect in a single dry cell with the binding posts B.′ The voltmeter indicates nearly 1.5, the slight drop being due to the fact that the cell must supply a little current to actuate the instrument. Attach wires at B and touch the tongue with them. The ammeter indicates, say .001 ampere. This shows a resistance offered by the tongue of 1,500 ohms—1.5 volts ÷ 1500 ohms = .001 amperes.

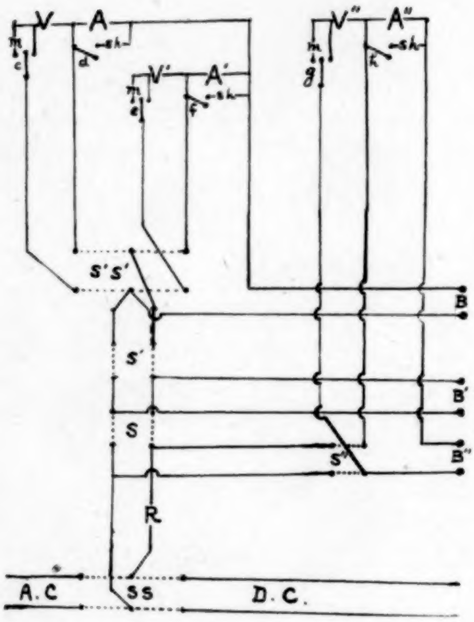

Fig. 1.

Experiment 2. Connect twenty-four dry cells in series at B.′ V′ now indicates 36 volts. Connect good metal handles with wires from B. Wet the hands and grasp the handles firmly. A′ indicates, say .01 ampere. The resistance was therefore 3600 ohms.

Experiment 3. Disconnect the battery from B.′ Close the switch S and throw the switch S S to the right connecting in the 110 volt direct current. Throw out all resistance at R. V′ indicates 110 volts. Touch the handles at B gently with the finger tips. A′ indicates, say .001 ampere. The resistance on account of the slight contact has gone up to 110,000 ohms. In-

crease the contact with the handles until A′ runs up to .01 amperes indicating a resistance of 11,000 ohms. With such variable resistance in our hands we may touch wires with voltages varying from 110 volts to 500 and judge of the quantities by the feeling.

It will be noticed that .01 ampere from the dynamo feels very different from the same amount received from a battery. All dynamos are alternating current generators and no commutator is able to make the current entirely continuous.

Experiment 4. Throw S S and S′ S′ to the left connecting in the 110 volt alternating current and the meters V and A. Now touch the handles as before. The quantities are the same as before but the sensation is different. Splinters or pin pricks lost or forgotten are discovered by passing the hand over these wires. For the execution of criminals 1800 volts and 10 to 14 amperes are employed. That quantity is obtained by special means for improving the contact.

Experiment 5. Open S. Throw S′ S′ to the right. Twist together copper and German silver wires at one end and insert the other ends into the binding posts B.′ The copper wire into the positive post. Short circuit the binding posts B. Heat the junction of the wires in a Bunsen burner flame. Cut out V′ by means of the switch *e*. A′ indicates, say .021 amperes or .025 volts for the same instrument measures both when the quantities are so small.

Experiment 6. By means of copper wires connect a rod of zinc and a rod of carbon to the binding posts B.′ Dip them into dilute sulphuric acid in a tumbler. With the circuit open at B, close the switch e: V′=1.4 and A′=0. Open e and short circuit the binding posts at B: V′=0 and A′=6. Close e: V′=.1 and A′=.3. Add sodium bicarbonate: V′=2. when A′=0. Chort circuit B: V′=4 and A′=2. Raise and lower slowly the rods in the solution. Note the effect upon the meters. Connect in a bell at B: V′=1.5 when A′=.15.

Experiment 7. With a plate of zinc and plate of carbon build up a dry cell and show its characteristics by the meters. In the same way try a Bunsen Cell.

Experiment 8. Connect a lamp socket at B. Put in various lamps in turn and connect first D C, and then A C, using the corresponding meters. Thus develop something like the following:

16cp. lamp $\frac{110 \text{ Volts}}{220 \text{ Ohms.}}$ =.5 amperes .55 Watts 3.5 Watt per cp.

Try lamps of 32 cp., 8 cp., 50 cp., etc.

Find how much current the Hylo lamp takes for maximum and for minimum brilliancy. Try the effect of a regulating socket. Ampere hours, Watt hours. Read a Watt meter and compute bills.

Experiment 9. Connect electrolytic cell at B. Use D C and throw in R. First put nothing but water in the cell. Note volts and amperes. Then add sulphuric acid and note the volts and amperes for suitable action. Note volume of hydrogen per Watt.

Experiment 10. Connect a storage battery first with B and note the characteristics of the charging current, and then connect it with B′ and with various pieces of a apparatus at B note the characteristics of the discharging current.

Experiment 11. Connect a rheostat at B. Use either D C or A C, and set R at say 220 Ohms. Note that by varying the resistance at B the volt meter may be made to read anything from 0 to 110. And the ammeter varies accordingly from .5 to 0. To illustrate by a water analogy. Connect a rubber tube with the water faucet and by means of a T tube connect a mercury pressure gauge at V′, see figure 2. At R put a Mohr pinch cock set

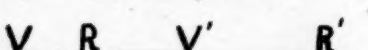

Fig. 2

so as to pass a very small stream of water. With the thumb and finger pinch the free end R′, more or less, so as to make the pressure gauge at V′ indicate varying pressures (in the case tried from 0 to 8 lbs. per square inch) when V′ was 0, A was a limpid stream dropping from the end of the tube. When V′ was nearly 8, A was a needle like stream spurting to a great distance. Of course in this analogy V stands for the 110 volt electric current, R for 220 ohms at the rheostat, V′ for the terminal voltage as indicated by the voltmeter, R′ for the variable resistance in the rheostat placed at B, and A for the variable current as indicated by the ammeter.

Experiment 12. By the method of experiment 11 adjust the terminal voltage of the 110 volt current so as to adapt it to any given miniature lamp. Several regulating sockets connected in series make a satisfactory rheostat for this purpose.

Experiment 13. To show the effect of temperature upon the

resistance of a wire. Use 100 inches of No. 24 iron wire. In the case tried V′ varied from 10 to 68 volts as the wire was passing from room teperature to red heat. At the same time A′ varied from 20 to 7.5 amperes indicating a resistance varying fram 1.5 to 10 ohms. A constant resistance of 4 ohms was kept at R to protect the instruments.

Experiment 14. Fuse wire warranted at 2 amp., carried 5 amperes for 10 minutes before melting.

Experiment 15. Connect a motor at B. Note the Watts and horse power required to run it without load. Note the same at full load.

Experiment 16. Take off the front of volt meters, ammeters and Watt meters. Show how like motors they are. Find resistance of voltmeters and amount of current required to actuate them. Find the resistance of the multipliers and show the function of multipliers for voltmeters and shunts for ammeters. Note that for very small currents the same instrument serves for measuring both millivolts and milliamperes.

Experiment 17. A certain magneto was shown to be an alternating current generator of 30 cycles, 50 volts on .035 amperes. It lighted a miniature lamp maintaining 40 volts on .1 ampere. But on the high resistance of the human body it ran up to about 100 volts on, say .001 ampere. While turning the crank the load of the miniature lamp was very noticeable.

Experiment 18. Find the efficiency of a motor-dynamo by connecting the motor end of the machine at B″ and the dynamo end at B′ and some apparatus to furnish load at B. If the motor is intended for alternating current, connect in A C; if for direct current connect in D C. V″ and A″ show what current it is taking. If the dynamo end of the machine is generating alternating current connect in V and A; if it is generating direct current connect in V′ and A′. Compare the Watts of energy consumed by the motor with the Watts supplied by the dynamo.

Experiment 19. Find the efficiency of a transformer by connecting the primary wires at B″, the secondary wires at B′ and some lamps at B to furnish loads. Compare the Watts as indicated by V″ and A″ with those indicated by V and A. Step up and step down the current.

The above 19 experiments may serve to indicate the type of work which I propose and furnish ground for a discussion which I hope to take up at some future time on the relative merits of lecture and laboratory work in the study of electrical measurements.

SUGGESTIONS CONCERNING HIGH SCHOOL BOTANY.

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To the teacher of laboratory botany, it is always something of a surprise to find that in many of the smaller high schools, botany, when taught at all, is still taught "out of a book." The reasons, or rather excuses, given for adhering to this out-of date method are various. Some stick to the book because they have no laboratory, some for lack of equipment, others because they have no suitable manual and still others because they imagine laboratory botany to consist chiefly in performing elaborate experiments with expensive apparatus.

As I regard the subject, laboratory botany in the high school consists simply in studying plant structures and plant processes at first hand, with experiments, when possible; without them when not. As such it may be carried on with great success in the ordinary class-room when the laboratory is not available. The teacher simply asks the questions he would in the ordinary recitation but substitutes specimens for the text and requires the pupil to get the answers from the plants themselves. In fact, botany, of all the high school sciences, is least in need of a laboratory. I do not mean to imply that one cannot do better work in a well appointed laboratory than elsewhere, but I do maintain that one need not give up botany by the laboratory method simply because he has no room fitted up expressly for its study. One needs a sharp knife and a sharp lens and it is astonishing what an adept can do with these. I am sure that if those teachers who are using the book could be prevailed upon to try the laboratory course for a single term, no matter how poor their equipment, they would not willingly go back to the old way.

Lest some should infer when I mention a pocket knife and lens, that the laboratory botany I have in mind is that which consists principally in "analyzing" flowers. I may add that with no better equipment than this, it is possible to complete a pretty careful study of seeds, buds, stems, roots, leaves, flowers, and fruits, in short, the usual high school course. I certainly do not underestimate the value of the compound microscope in botanical work, but I would emphasise the fact that this instrument is not absolutely essential. If the school has a single instrument, through which the pupils may view each structure studied, so much the

better, but if not, a good lens, costing less than two dollars, will enable the student to see as much of the minute structure of plants as a high school pupil needs to see. To get results, however, it is necessary to prepare the specimens to be examined exactly as they would be for the microscope. They mut be cut very thin, mounted on a slide in water or other media, covered with a cover-glass and viewed against strong light, with the best magnification possible. By this means one may see stomata, root hairs, all the regions of the fibro-vascular bundle and need not omit even a study of cells. In making thin sections a sharp razor will answer in place of a microtome, the object to be sectioned being held between two pieces of elder pith.

Without doubt one of the greatest drawbacks to the progress of laboratory botany in the small high school, is the lack of a suitable manual. Those in use at present do not contain questions that are direct and definite enough. In work with college students, it may be well enough to tell them what to look for and how it is arranged with regard to neighboring structures, but if you tell the high school pupil this, he will usually take your word for it and miss the invaluable drill that comes from discovering things for himself. If we hold that botany in the high school course is as valuable for the training in observation and deduction it gives, as for the facts it imparts, surely is is far better to ask questions that can be answered only by a careful study of material. Until a book containing a complete set of such questions appears, teachers will have to work out such question for themselves. It is not to be expected that a new teacher will make finished outlines during the first year, but he will not make the same mistake twice and soon his outline will take on accuracy and completeness.

The materials for laboratory botany are usually easy to obtain and, once collected, a great part may be preserved and used again and again. In collecting, the nearest woodland, thicket, river-bank, old field or even the corner grocery, will prove a fair substitute for the botanical garden. If there is a school garden accessible, it is so much good fortune. For the benefit of those teachers who may desire to take up the work, some of the materials and their uses may be mentioned.

The main topics to be kept in mind in the study of seeds is their structure, and the condition required for germination. Corn and beans, the bigger the better, are as good as any to show struc-

ture, one being typical of monocotyledons and the other of dicotyledons. Peas, pumpkin seeds, castor-beans, and honey-locust seeds are desirable for comparison, but not necessary. A few flower-pots and a window in which seedlings can be grown are also desirable.

The study of roots is concerned with their structure and uses. Tap-roots, such as carrot and parsnip for the study of their gross anatomy may be obtained at the grocery, and any roadside will yield other examples such as the shepherd's purse, dandelion, etc. Cuttings of Tradescantia ("wandering Jew") rooted in water will give good root-caps and root-hairs and the nearest corn-field will supply prop-roots. The ubiquitous poison ivy may be depended upon for aerial roots if the trumpet creeper or English ivy is not to be had.

The entire study of buds may be made out of doors, if the weather permits and in any event, a field trip is desirable. Almost any thicket will supply the materials. If naked buds are not found, look elsewhere for butternut, witch-hazel and pawpaw. The buds of the butternut are invaluable for showing an unusual placing of the accessory buds while maple or peach will show the commoner form. The leading types of bud protection will be illustrated by sumach, Ailanthus, cottonwood, smilax, button-wood, horse-chestnut and lilacs.

The essential differences between monoctyledons and dicotyledons may be the main line of attack in the study of stems and with this may go a study of cork and its uses, the location of the paths for the transfer of foods and food materials and an investigation of the various modifications of stems and their uses. The common garden asparagus or corn, cut while green, and preserved in alcohol or formalin, shows the monoctyledon type of stem well, while Begonia, balsam, Geranium or sunflower are good for the dicotyledon type. If microscopes are at hand the pupils may make a study of the cells in the different tissues; if not, he can at least see the ducts with a simple lens and locate the other tissues. For a study of lenticels, sumach, elder, birch or cherry are good. Specimens of bulbs, corms, crowns, tubers, rhizomes, cladophylls, thorns, tendrils, etc., must be collected as chance offers and preserved in alcohol. Some may be obtained of the seedsman or florist. Good examples of the types named in their order are as follows: onion, hyacinth or lily, crocus or gladiolus, parsnip or carrot, potato or artichoke, Solomon's seal or blood-root, as-

paragus or the smilax (Myrsiphyllum) of the florist, hawthorn or osage orange, grape, woodbine, or Boston-ivy. The hop, bean or bitter-sweet will show twining stems and the rose, bedstraw or blackberry will illustrate clamberers. Sections of oak, ash or other trees will be needed for showing the annual rings and young twigs of basswood or lilac are invaluable for work on the regions of the stem.

In leaves, again, the differences between monoctyledons and dicotyledons are to be emphasised as well as the different types of venation in each. The two forms of branched leaves in the dicotyledons corresponding to the two types of venation may be included, also. The relations of leaves to light, their arrangement upon the stem, their devices to avoid shading, etc., may be studied in the class-room as easily as in the laboratory, but study in the field when possible is better still. Little can be done in the minute structure of leaves without the microscope, but a study of the epidermal outgrowths, such as simple and glandular hairs, scales, bloom, etc., may be carried on with a lens and also the various modifications of leaves such as thorns and tendrils. Only leaves showing the less common forms may be mentioned here. Canna, banana and Calla will show transverse veining in monocotyledons and the lupine, easily grown from seed, or horse-chestnut will give palmately branched leaves. The Easter lily or some of the narrow-leaved Dracænas are excellent for comparison with rubber plant to show the relation between breadth of leaf and the number of leaves on the stem. The house-leek, dandelion or teasel will afford fine rosette-plants and the leaves of the mullein, geranium, primrose, Sheperdia and hickory will give interesting forms of epidermal hairs. Sections of rose or gooseberry, locust or prickly ash, barberry, smilax, pea or vetch, sundew or pitcher plant, carnation or cabbage will be needed to show various modifications of leaves and epidermal structures. The pitcher plant is easily obtained from nurserymen and may be grown in the school room all winter. For cross-sections, the leaf of rubber-plant is suggested because all the tissues are well defined.

The study of the flower can be completed in the field with spring classes, but autumn classes will have to depend upon the greenhouse or florist. Scilla and wild hyacinth (Camassia) are the very best of monocotyledons types, but tulips will answer. The Trillium, so often suggested is not to be recommended because the leaves are not typical of monoctyledons. Gamopetal-

ous flowers should be avoided in the first study, if possible. Any simple type of dicotyledon flower will do. The best is some member of the live-for-ever or house-leek family, but these are not always obtainable. The butter-cup, Oxalis, Geranium or cherry may be substituted. Any adequate study of the forms of flowers and the uses of the parts in effecting pollination must be made in the spring with flowers in the field.

Oranges, apples and beans will show all parts of the flower that ordinarily enter into fruits. In this study, however, the teacher may have greater variety if the pupils are allowed to select their own materials which may be eaten as soon as the pupil can name the parts of the flower which each represents.

If care is taken to see that the written work in answer to the questions given is neatly and accurately done, and if sufficient drawings of the various structures are made to ensure that they have been properly seen, it is certain that students taking such a course will leave it better acquainted with botany and with a greater love and respect for the subject than any pupil who got his botany "out of a book."

AN APPRECIATION OF THE WORK OF LUTHER BURBANK.

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Luther Burbank was born of English-Scotch parentage fifty-eight years ago last March in the little town of Lancaster near Boston. He completed the common school course and for several years during the winter term only, attended the local academy. The rest of the time he worked as a factory hand chiefly at Worcester. As soon as his savings would permit, he began work as a market gardener, impelled to this line of work by an extraordinary love for plants. He began experimenting at once and in the early '70's had brought out several new varieties of potatoes, one of which was the well known Burbank potato. In '75 he sold his stock of the Burbank potato except a few tubers for $125 or $150 and went to California, where he found better conditions of climate both for his health and his work. "Out of doors," in California, he found to be better than a greenhouse in Massachusetts. He settled at Santa Rosa, about fifty miles north of San Francisco.

After thirteen years of struggle he purchased his present farm of fifteen acres at Sebastapol, eight miles from Santa Rosa and

established a large nursery that soon brought him an income of $10,000 a year. In 1890, he sold his nursery business in order that he might devote himself exclusively to the work of plant improvement. (De Vries).

During his experience as a gardener and nurseryman, he acquired a varied and valuable skill in the manipulation of plants. To a remarkable ability to make plants vary were added an equally remarkable range of sense perception, memory of form, flavor, odor, color and an extraordinary power, amounting to genius of recognizing the correlations of the characters of plants and their value in predetermining the nature of a plant while yet a seedling. A similar recognition of correlations is characteristic of Nilson's work in Sweden and is accompanied by a careful cataloging of them.

Mr. Burbank's work has been exploited to such an extent by newspapers and some of the magazines that he and his work are looked upon by many scientific botanists with very great prejudice. Although he has not the scientific training of the schools, he is a great scientist for all that. It may be admitted that Mr. Burbank has made no additions to the known laws of science as stated by Dr. Vernon L. Kellogg. It does not follow that his work is without interest to the pure scientist. He has been thoroughly occupied with the practical side of plant breeding. His work, however, meets with the highest respect from those pure scientists who are best acquainted with it. He is "the foremost practical plant breeder in the world," says Dr. De Vries.[1]

He adds farther, "It requires a great genius and an almost incredible capacity for work together with a complete devotion to the purpose in view to accomplish such results. Burbank possesses these qualifications."[2]

Dr. R. S. Woodward says, "Mr Burbank has done capital work and is still engaged in productive enterprises. . . . It will probably be two or three years before we complete our investigation. The work is as difficult as it is important." Dr. De Vries, in search of material in support of his doctrine of mutation, has, in his two visits to Mr. Burbank, found much material encouragement. In speaking of the extraordinary range of variation,[3] he says:

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[1] *Pop. Sci. Mon.*, 67-329.

[2] Ibid.

[3] *Pop. Sci. Mon.*, Oct. '06.

"The result is that many thousands of seedlings are required to go beyond the ordinary range of variations. . . . . Hence the rule that great results can only be attained by the use of large numbers. . . . . It is to Luther Burbank that we owe this great achievement. His principles are in full harmony with the teachings of science. His methods are hybridization and selection in the broadest sense and on the largest scale."

In illustration of this statement the following facts are quoted in part from an article by Dr. Vernon L. Kellogg, Stanford University, and in part from an article by Dr. Hugo De Vries.[4] Mr. Burbank carries 300,000 varieties of prunes at one time by grafting. The white blackberry was one plant selected out of 65,000. All but the one were burned.

Out of 40,000 dewberry and raspberry hybrids "Phenomenal" is the only one now in existence. From 500,000 lily bulbs 50 were reserved for further cultivation, the rest destroyed. From 10,000 to 15,000 rose bushes, three good varieties were chosen the rest burned. "In one year he sent to the rubbish heap 400,000 seedlings of hybrid plums."[5]

Mr. Harwood states that Mr. Burbank has had as many as 2,500 species of plants under experiment. "The magnitude of Burbank's work excels anything that was ever done before, even by large firms in the course of generations."[6]

In speaking of current "Plant Breeding Practice," I think L. H. Bailey has Mr. Burbank in mind. He says: "There are persons who have unusual native judgment as to the merits and capabilities of plants and who develop great manual skill, but they are plain and modest citizens, nevertheless, and their methods are perfectly normal and scrutable. The wonder-mongers are the reporters not the plant breeders."[7] Later, in speaking specifically of Mr. Burbank among plant breeders, he says: "Yet, if there are other plant breeders. Luther Burbank stands alone. . . . He is a gardener of a new kind. . . . pursuing his work in his own way and because he loves it so well that he can not forego it. . . . He grows everything he can, no matter where it comes from or of what kind. He cultivates with personal care, multiplies, scrutinizes every variation, hybridizes indiscriminatingly, saves the seeds of the forms that most appeal

[4] *Pop. Sci. Mon.*, '07, 331.
[5] *Current Literature*, 39, 213.
[6] *Pop. Sci. Mon.*, '07, 346.
[7] *Plant Breeding*, p. 228.

to him, sows again, hybridizes and selects again, uproots by the hundreds and thousands. . . . . and now and then saves out a form that he thinks worth introducing to the public. . . . Every form is interesting, whether it is new or the reproduction of an old one. He shows you the odd and intermediate and reversionary forms as well as those that promise to be of use to other persons. . . . The methods are the common practices of all plant breeders, made unusually efficient, perhaps, by the geniality of the climate, the great scale on which some of the work is conducted, the wide variety of plants under experiment and the patient skill and good judgment of the man. He cares little for the scientific method so long as the plants produce new forms. He will try to cross anything, no matter whether it has ever been crossed before, or whether the crossing is in utter disregard of all botanical relationships. Once I asked him the botamical name of a plant. He replied that he did not know and did not care to know; for if he knew, he would likely be bound by the book statements and he might be handicapped in his work. He is a bold worker and this accounts for some of the odd results. . . . the value of Mr. Burbank's work lies above all merely economic considerations. He is a master worker in making plants to vary. Plants are plastic materials in his hands. He is demonstrating what can be done. He is setting new ideals and novel problems."

Other scientists of high standing might be quoted in the same vein, but enough has been said to indicate that the time is not yet come for a proper appreciation of the real value to pure science of Mr. Burbank's work and that the proper attitude of biologists meantime should be one of respectful, hopeful expectancy.

The author of a recent article on the interchange of heat in steam-engine cylinders takes the example of a steam engine running at 300 revolutions per minute, and argues that, as the entire stroke is covered in one-tenth part of a second, there is not time for any interchange of heat between the metal and the steam. Experiments said to be analagous are quoted in support of this view of slow interchange of heat, and the author concludes that it is the water of condensation acting directly on the entering steam which is the real agent of the so-called interchange of heat losses, and that this action is greatly assisted by the free mixture of the products of compression with the entering steam.

LONG RANGE FORECASTS.

The following extracts from the *Weather Review* of March, 1906, and February, 1907, are interesting in that they show the attempts that are being made to extend the period of forecasting, and the hopes that are entertained for the future. The article on South African forecasts bears the initials of Cleveland Abbe, editor of the *Review*:

Long Range Forecasts in the United States.

The following statement has been sent by the Chief of the United States Bureau in reply to a recent letter, requesting some details regarding the "new departure in forecasting weather conditions a month in advance:"

"Beyond the statement made by me in New York in March, that the Weather Bureau believes that it is in possession of a sound scientific basis on which to make forecasts for a considerable period in advance, nothing will be announced in regard to the matter for several months to come."

Indian Monsoon Forecasts.

The annual publication by the director of the Meteorological Service of India of a statement of general atmospheric conditions, with an attempt to forecast the general character of the southwest monsoon rainfall, has now proceeded for about twenty years with a variable degree of success, but sufficient to show that the effort at long range forecasting is really worth while. The work was begun by Blanford, was carried on by Sir John Eliot, and is now in the hands of his successor, Gilbert T. Walker. Pending a more extensive investigation into the philosophy of these seasonal forecasts we quote the following remarks from a review of the subject by Professor Hann, of Vienna, as published in the *Meteorologische Zeitschrift* for February, 1907.

Blanford thought that he had shown that generally snowfall in the regions to the north and west of India produced an abnormal distribution of pressure over northern India that was unfavorable to the advance of the southwest monsoons over this region; and he adopted the general principle that lower atmospheric pressure over any area increased the amount of its rainfall.

Sir John Eliot showed that the conditions over India alone would not suffice to justify reliable forecasts, and after the

year 1894 information as to the conditions over the Indian Ocean was made use of, extending annually farther south, until, in 1897, even Africa and Australia were considered. It seemed most probable that heavier rainfall at Zanzibar and off the Seychelles in May would justify predicting heavier rainfall in India later in the season, when the monsoon has crossed over the equator. But later experience showed that the opposite was the case. Then it was assumed that perhaps an abnormal high pressure over Mauritius would increase the monsoon current and the Indian rainfall; but this did not prove to be the case. The relations between rainfall in India and atmospheric pressure over India, Siberia, the Indian Ocean, and South America are such that in years of excessive monsoon rainfall in India, the atmospheric pressure over South America is too high, and conversely, small rainfall in India goes with low pressure in South America. Moreover in many years the low pressure occurs earlier in South America than the small rainfall in India. A table of snowfall for fifteen years shows that in the case of heavy and late snowfalls, when the area in the Indian highlands covered with snow in May is larger than usual, it argues for small rainfall in June.

Thirteen years of records show that heavy rainfall in the subequatorial region, over Zanzibar and the Seychelles, brings deficient rainfall in India over both branches of the monsoon: so that in general the snowfall in upper India is not connected primarily with the subsequent defect in rainfall, but is only an indication of a disturbance in the general circulation of the atmosphere. Moreover, excessive rainfall at Zanzibar in April and May coincides with deficient height of the flood wave in the Nile River; so that we may say that a deficient snowfall in upper India coincides with a deficient flood in the Nile. On the other hand heavy snowfall in India and heavy rainfall in the equatorial region is paralleled by the connection between abnormal rainfall at Zanzibar and the Seychelles in November, with heavy snowfall in upper India in the subsequent cold season.

With regard to atmospheric pressure and rainfall, high pressure in Mauritius means small rainfall in India, and low pressure in Mauritius is followed by heavy rainfall in India, in a large majority of cases, namely, 80 per cent. Comparing pressures in Argentina with rainfall in India, Walker and Hann are led to the remarkable result that positive departures of pressure in the

spring at Cordoba are followed by positive departures of the next following summer rainfall in India. The worst drought in India, with a rainfall departure of —24 per cent, or 254 millimeters, was preceded by a departure of —1.4 millimeters of barometric pressure at Cordoba; whereas the best monsoon rainfall, 1892, with a departure of + 124 millimeters, was preceded by + 1.8 millimeters of pressure departure at Cordoba. According to the last memorandum by G. T. Walker, large departures of pressure in July at Mauritius have a close connection with the simultaneous opposite departures of rainfall in August and September over the whole of India.

In conclusion Hann says that the method by which Walker has carried out his investigation seems to be the most appropriate, namely, the comparison of the departures of the meteorological elements. We must know both the direction and the quantity of the departures. We must compare them geographically as well as chronologically. The best example of such work consists in Hann's study of the anomalies of the weather in Iceland as compared with those on the continent of Europe.

Seasonable Forecasts for South Africa.

Mr. D. E. Hutchins, conservator of forests for South Africa, read a paper before the South African Philosophical Society, at Cape Town, November 29, 1905, entitled, "The cycle year 1905 and the coming season." An abstract, furnished by Mr. Hutchins, occupies pages 98-105 of the Agricultural Journal of the Cape of Good Hope for January, 1906, Volume 28, No. 1. The author has made an elaborate study of the rainfall data published regularly in the Agricultural Journal by Mr. Charles M. Stewart, Secretary to the Meteorological Commission of Cape Colony; and while his first thought has been to work out chronological cycles in South African rain, he has also looked for geographical relations, namely, the relations between the rains of South Africa and those of the states to the north of it, as well as of more distant countries. The first publication of Mr. Hutchins that we find mentioned is one of 1888, entitled "Cycles of Drought and Good Seasons in South Africa." For the present paper of November, 1905, he prepared diagrams of the rainfall records at Cape Town, Grahamstown, and Durban, which accord remarkably with the three cycles that he has worked out for Cape Colony weather. These cycles he designates as follows:

(1) The "solar cycle" of 11.11 years, whose maximum oc-

curred in 1905, and which happened that year to agree with the maximum of sun spots, but does not always do so; three of these solar cycles, or the 35-year sun-spot cycle, he calls the "Brueckner cycle."

(2) A cycle of alternating periods of nine and ten years, or an average of nine and a half years; this he terms the "storm cycle;" its maximum will occur in 1907; it has its greatest influence on the winter rainfall of the western portion of South Africa, while in the eastern portion it is liable to bring only wind. This also corresponds to the mean period between successive droughts in Australia.

(3) A cycle with alternating periods of twelve and thirteen years, or an average period of twelve and a half years; this he terms the "Meldrum cycle," in compliment to that eminent meteorologist; this cycle brings a good deal of general rain, but usually especially affects the summer rainfall of the eastern districts of South Africa.

These three cape weather cycles were recognized by Mr. Hutchins in 1888. By means of them the rainfall of that year was predicted, and he states that they have agreed with the rainfall of subsequent years, with but few failures. After a detailed consideration of the rainfall for each year from 1865 to 1905, he states the following conclusions: (1) The three main weather cycles are of general application throughout South Africa, and the storm cycle and Meldrum cycle are of general application east and west beyond their areas of greatest influence. (2) As we go northward the heavier rainfall occurs a season earlier. (3) There are obscure indications of a tendency to rain at the sun-spot minimum, but the normal minima (he uses 1844-1855-1867-1878-1889-1900) have so frequently coincided with the other cycles that the exact influence of the sun-spot minimum is difficult to trace, and further observations are necessary. (4) Up to the present time the direct influence of Brueckner's 35-year cycle is inappreciable in South African weather.

In conclusion, Hutchins offers a forecast as to the rainfall. In 1888 he published a forecast for the year 1905, namely, that the sun-spot cycle and the Meldrum cycle would coincide, and therefore "most probably general good rains." For 1906 his prediction then was: "Probably good rains, with drought at a few stations." He claims that the forecast for 1905 was well

verified, and now, namely, in November, 1905, after studying recent conditions, he offers the following forecast for the next two years:

"For South Africa generally, except the southern and southwest coast of Cape Colony (all of which is included in his summer rainfall area), the year 1906, coming between two rainfall periods, may have short and local droughts, or the rains may run on to the heavy rainfall period which is ahead of us in 1907, and probably in 1908; the outlook now is for several years of good rainfall ahead.

"For the south and southwest coasts of Cape Colony strong 'southeasters,' really southerly and southwesterly winds, may be expected during the summer; the cyclical indication for next winter's rains (1907) is that they will be moderate.

"Long-period forecasts cannot have anything like the precision of the short, day-or-two forecasts; * * * they are at best but a calculation of probabilities and an indication of what may be expected to affect the coming season as a whole. * * * For the drier inland districts the rains are too irregular for the cyclical forecast to have any practical value. After 1908 there are six years of drought to be looked forward to, with an irregular mitigation of the drought, most probably about 1911 or 1912."

The more we consider the statements contained in Mr. Hutchins's abstract, the more certain does it seem that he has shown that sun-spot cycles have nothing to do with the variations of rainfall in South Africa. On the other hand, he has shown that there are certain correlations between the rainfall in the east and the west, such that when the one goes up the other goes down. He finds so many exceptions to the chronological regularity of heavy and light rains that the probability is that there is no such regularity at all, so that cycles of 12.5, 11.11, 35, and 9.5 years, all of which he investigates, have no real existence. But, on the other hand, the fact that the area of heavy rain, with its outlying region of lighter rains, moves about, east and west, north and south, becomes clearly evident. When this geographical motion is large one is tempted to hunt for a cycle to correspond, but nothing of the kind appears from his data. On the contrary, by studying the monsoon data of the east coast of Africa it is plain that as he himself says, "the rains are too irregular for the cyclical forecast to have any value."

THE TEACHING OF BIOLOGY IN SECONDARY SCHOOLS.[1]

BY J. F. ABBOTT,

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We hear a good deal nowadays of the "exact sciences" and the "inexact sciences," and of the superiority of the former in secondary school work, or, as more concretely understood, of the much higher educational content of Physics and Chemistry in contrast to Biology.

It cannot be denied that Biology as frequently taught is "inexact" to an extreme degree nor that the training and discipline involved in Chemistry and Physics is usually of a higher order than that of such a wholly descriptive science as Animal or Plant Morphology. But is this condition necessarily incapable of improvement and is the movement that seems to be gaining favor in some quarters of crowding Biology out of the curriculum, to give space to more practical matters, justified? Most of us, I imagine, disagree with such a conclusion, but few of us will deny that there is room for improvement.

The educational value of the study of Nature, animate or inanimate, is now definitely acknowledged. To know the birds and the woods, the ways of beetle and butterfly, fern and flower, and the infinitely delicate adjustment of living things to the world about them, surely such as that is a liberal education in itself and one that is too seldom the heritage of the city-bred boy or girl. But the attempt to teach such subjects successfully in an ordinary secondary school must often be disheartening to the most enthusiastic teacher.

I do not disparage the work and influence of those often unknown and unheralded lovers of Nature who, with the spirit of Agassiz, interest and open the eyes of even their dullest pupils and really vitalize their "Nature study." But such, unfortunately, like poets, are born, and not made in Normal schools. It is hardly to be wondered at that a serious study of biological nature should so often be unsuccessful among High School students, when we think of the difficult conditions under which it is attempted. It is a great misfortune that every High School boy or girl cannot roam the fields in his biological studies. The ap-

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[1] Read before the Biology Section of the Central Association of Science and Mathematics Teachers in St. Louis, Nov. 30, 1907.

preciation of the need for and of the value of such natural history studies, together with the reaction against dissections and minute anatomy has led to a return in many quarters, to the text-book recitation system and many most excellent text-books have been produced to supply this need. But this, I fear, has turned out too often, in the hands of mechanically inclined teachers, to be a cure worse than the disease. It is saddening to see a class of boys and girls reciting on, say, thirty pages of "Jordan and Kellogg" with no more consciousness that the subject matter of their lesson is real and now-existent than they have in a class in Egyptian History.

It would seem that Natural History is no more of a success in the class-room recitation than in the more superficial and somewhat amateurish outdoor work; on the other hand there is an almost universal concensus of opinion that the working out of the minute anatomy of a series of animals, a la Huxley, has no place in a secondary school. Evidently a new interpretation must be found for Biology.

The word "Biology"is one of the few that always recall our scattered classic derivatives and were I to ask any teacher its meaning there is no one so unlettered as not to be able to inform me, but if I were to ask what Biology as a school study is, I should discover that according to current curricula it means "a half-year of Zoölogy plus a half-year of Botany." For this definition we have the authority of the college catalogues, or at least of those sections devoted to entrance examinations, and it is the usually accepted status of the study in most secondary schools.

If Biology by its derivation means the "Science of Life," or the "Study of Life," it surely merits the most prominent place in any curriculum. Perhaps the reason that it does not do so may be that as taught nowadays it is sadly misnamed. A half-year of zoölogy, that is, animal morphology, plus a half-year of the same kind of botany, does not compass the "Science of Life." It is true that our knowledge of the nature of life processes is most hazy and hypothetical in many subjects even now. Yet little as we know of the laws of the actions of living things and their interactions on each other, our conceptions of such phenomena have broadened and deepened in a way incredible to one who has not kept pace with investigations of the past ten or fifteen years. Apparently the time has now come to consider whether we can apply this newer knowledge successfully in the field of secondary school work.

It is interesting to observe how our school studies follow in a general way the path of research and progress, but always a decade or so behind. Biology used to be "Natural History" synonymous with Linnean zoölogy and botany. When these studies had advanced to the field of morphology our school-boys were still analyzing flowers and learning how the various groups of mammals are distinguished by their dental formulae. Gradually morphological studies displaced systematic ones in the schools and the study of zoölogy became Comparative Anatomy, though frequently with but slight accent on the "Comparative." Meanwhile research and the lines of scientific progress have been moving out of the field of Morphology. In the paths in which they are leading we will find it increasingly difficult to follow in secondary school work—at least so far as fundamental experiment or laboratory work is concerned.

It has thus become more and more a question whether it is not advisable to drop out the secondary school biology and be content with the nature-study of the grades, leaving the newer biology to be taken up in college in a much more intensive and extensive way than is possible in a high school. But we are met here with the difficulty that the high school is not or ought not to be a College Preparatory School, and if Biology is as important a subject as we think, the student who never gets beyond the High School ought not to be deprived of the opportunity of studying it. In order to understand this situation, let us consider for a moment what the newer Biology involves. We find first that there is much less interest manifested nowadays in the mere static form and structure of animals or of plants. On the other hand there is a good deal of interest in the changes in form and structure as animals or plants adapt themselves to a changing environment, and the study of such causes and effects, together with the accumulation of a mass of data on the subject has given rise to a special branch of Biology which we christen Ecology, and which we may think of as Nature-Study codified. Such a study involves a consideration of species and it is interesting to note how generally Biologists are turning back to a branch so largely neglected during the latter third of the past century. But here again it is from a different point of view that species are considered; as dynamic factors and not as fixed static entities. In the study of development, cytologists and embryologists have traced the elements of the cell through their various phases of

change until a wholly new light has been thrown on the subject of heredity in the individual. At the same time chemists have analyzed and then attempted to synthesize the complex living substances called "Protoplasm," in which life resides, and find no new substances, no different methods of combination and no laws applying that do not also apply to the chemistry of non-living matter. A great impulse has been given to the science of physiological chemistry—which should be called Biological chemistry. The chemist again, as physiologist, has been tracing the ultimate course and fate of the food substances taken into the organism and the more complex chemical processes of metabolism within the protoplasm itself by which energy is released or stored up. The physicist, like the chemist, finds that protoplasm obeys no different laws, indeed betrays no different type of structure than that familiar in the non-living world. The experimental Morphologists are showing in their work on Regeneration both in egg and adult, in their breeding experiments, and in their observations of animals and plants under altered conditions that both individual and race must be considered as plastic organisms, responding in an infinitude of delicate adjustments to every change in the external world.

Without any intention of giving a resumé of the field of modern Biology, I wish merely to emphasize two facts. First, that Biology to-day is a study of natural forces, of Dynamic Nature, whilst that of two or three decades ago was a study of static nature; and second, that whereas Zoölogy and Botany not long ago were isolated branches having little connection with other sciences, or with each other, nowadays not only does the Botanist, if he is wide awake, find it impossible to avoid trespassing on the territory of Zoölogy, and vice versa, but it is equally impossible for both to get along without the help of Mathematics, Physics, and Chemistry. Biology has become more of a unified science than ever before. Our problem is: Can we introduce such a complex science to secondary school pupils, and if we can, is it desirable?

I believe that from at least three points of view such a study is desirable for every High School student.

First: The Educational or Philosophic content.—Every student should be impressed with the fundamental unity of nature and with the idea that the same laws and principles underlie the life of all living things.. To this end there should be no

Botany or Zoölogy or Physiology as separate pigeon-holed sciences, but animals and plants should be used to *illustrate* the workings of such fundamental organic laws. I wish especially to emphasize this point as it is almost diametrically opposite to the method of procedure employed nearly everywhere in the teaching of Botany and Zoölogy. Whenever the Plant world offers better illustrative material let the experiments and illustrations be Botanical, or contrariwise. Best of all, let both be studied together. To give a single concrete example: When a student has observed a frog in a bottle containing hydrogen instead of oxygen, but with a lime-water indicator attached, give off CO_2 when stimulated to movement by electricity, proving that the CO_2 is the direct product of the destructive metabolism of muscle and has no connection with atmospheric oxygen—and at the same time, on another table, observes that peas swelling in a test-tube of mercury (the familiar experiment of so-called intra-molecular respiration) give off the same gas and displace the mercury in the test-tube, he not only has gained an insight into one of the profound problems of modern Biology, but he realizes, too, that the problem is the same for both plant and animal.

Second: The sort of training involved.—Nothing can compare with animal or plant morphology (unless it be drawing) in developing the "seeing eye" as it has been aptly called, the ability to look at a thing and see it in its relations and details. Moreover, the consideration of kinds of plants and animals leads to a sense of logical arrangement and system, and the study and comparison of *species* (an important phase of Biology nowadays almost obscured by the predominance of Morphology) is the best thing to develop judgment of which I know. At the same time the performance of laboratory experiments involving Chemistry and Physics, general Physiology, in short, cannot but develop clear. inductive thinking. This, too, is especially true in any really scientific study of animal behavior, or of the adaptations of organisms to environment, for here we must follow the scientific method closely and draw our conclusions from as large a mass of data as possible.

Third: The practical relation of such a study to everyday life.—To mention two or three examples out of hundreds: If the facts of animal parasitism and disease transmission were a little better disseminated, we would not have such Mediæval ab-

surdities as shot-gun quarantines in times of yellow fever epidemics and the pest of mosquitoes and of malaria by intelligent united effort could be eliminated in a few years. If the facts of the method of growth and development of bacteria were better known we would not have the scourge of typhoid fever epidemics that come so frequently. These things are in point of fact phases of General Biology, and if the general principles of the life history of the myriads of germs, so-called, in the midst of which we live and move and have our being were studied and taught in our public schools in a really scientific way we must go far to find anything more truly practical. For the facts of General Biology constitute the scientific basis of Hygiene.

I think I already hear objections that this is all very fine, but that such a course is impracticable, visionary and incapable of organization—that to attempt to teach such a course would be to expect too much from both teacher and pupil. Obviously, since a preliminary knowledge of both Chemistry and Physics is assumed such a study must come late in the High School Course, preferably in the last year.

During the past three years a course has been given in Washington University on the plan just outlined, which has been elected largely by Freshmen in the College. And as College Freshmen and High School Seniors are not so dissimilar that the same conditions do not apply fairly well to both, we may feel justified in believing that whatever measure of success, or practicability, may have been attained in the one case might also be expected in the other. In lieu of a discussion of the subjects to be included in such a course, or of the manner of handling them, may I be permitted to describe the course just referred to—not with any pretention to its being even approximately ideal nor even wholly satisfactory, but because it has been found workable under the conditions mentioned.

It is self-evident that without some knowledge of both Plant and Animal Morphology a student could not do very clear thinking on physiological matters. Accordingly, the greater part of the first half-year is taken up with a study of plant and animal morphology. The doctrine of "Types" has *not* been adhered to and the work is made both comparative and ecological. The course begins with a study of various kinds of ferns. In connection with the structure and function of the fern leaf a comparative study of other leaves is undertaken, and in connection

with the fern rhizome a comparative study of all sorts of stems. This is followed by a general study of the root. In all cases much attention is given the effect on form and structure of various changed conditions of moisture, dryness, heat, soil, etc. This is followed by a study of the comparative external anatomy of wasps, beetles, grasshoppers and crayfish, together with lectures or talks on the habits and life histories of these animals. In addition, the internal anatomy of the lobster is taken up and a certain amount of attention given to such general morphological matters as symmetry, metamerism, etc., and a great deal of attention to the comparative morphology of the anthropod appendages.

That is followed by a further study of various plants higher and lower than the ferns, and this in turn by a rather detailed study of the frog as an example of the vertebrate type of structure.

Reproduction in the Plant Kingdom is then taken up and a comparative study made of the life history of the fern and of other plants. Then the structure of the cell, the segmentation of starfish and of frog eggs, and the superficial outlines of the development of the chick, using living eggs. In connection with cell studies the unicellular infusoria are taken up and this leads to the subject of the Bacteria, their culture, the preparation of media and methods of staining, and a discussion of the preparation and use of antitoxin. This again leads to the consideration of fermentation and yeast, and this to the study of aerobic and anaerobic respiration (energesis), and protosynthesis, and the action of enzymes in general, fat splitting, proteolytic, sugar splitting, etc. The working out of laboratory experiments in this subject is far easier than might be supposed. This is followed lastly by the physical phenomena of protoplasm, osmosis, imbibition, absorption, etc.

The rest of the course is given over to field work and laboratory work combined, and in connection with this the students are introduced to the subject of variation in Animals and Plants, working out for themselves the curves of variation and correlation of various natural objects, such as leaflets of plants, spines of grasshopper legs, dimensions of beetles, etc., each student having an individual problem. In connection with this the outline of the Theory of Natural Selection is introduced.

The student is required to take care of the material he brings in from the field, and by means of it is made acquainted with

the general system of classification of Animals and Plants. Not only does he "run down" his specimens in keys, but, what is considered much more important, he is given a number of related forms, e. g., the various common grasshoppers, crickets, katydids, and is required to construct descriptive keys of his own.

I realize the apparent crudeness and apparent lack of continuity in the outline just presented, but in practice no difficulty is experienced in leading from one subject into another and by the end of the year there have been outlined the fundamentals of general morphology, embryology, bacteriology, general physiology, ecology, variation and the data of the theory of descent.

When the student has finished such a course we hope that he has a greater interest in the world in which he lives, and a better understanding of the natural forces that operate in it, but more than that, we believe that his curiosity has been aroused and his powers of observation sharpened so that he will, on the one hand, be moved to find out the answers to the problems that confront him everywhere, and, on the other hand, will depend on his own senses and judgment in working them out.

To summarize: I believe that the teaching of the subject of Biology, which in its broad sense is one of the most important studies a secondary school student may take, needs to be adjusted to the great advances that have been made in recent years, or else lose its place in the secondary school curriculum; and that the discredit that has come to it as a High School study is due largely to the somewhat dilettànte and superficial way in which it has been given. Biology is a fundamentally deep and broad subject, frequently too much so for the peace of mind of the average teacher, and is capable of as much intellectual discipline and important information as either Physics or Chemistry. It should by all means be a presentation of the dynamic aspect of the science rather than the static, and it should be made broad enough to cover the whole field, doing away with the distinctions involved in Botany, Ecology, Physiology, or Zoölogy. There is no reason to doubt that such a course is not only desirable, but practicable; but it goes without saying that teachers must be fitted by proper training to conduct such work.

A DISCUSSION OF THE REPORT OF THE COMMITTEE ON GEOMETRY.

By G. C. Shutts,*
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After studying the report, one feels that it must meet with general approval among teachers in nearly every particular. The statement that it is "meant to be evolutionary not revolutionary" suggests the temper of the movement. It appeals to the good sense of teachers and asks them to co-operate in the suggested improvements as far as their judgments endorse any changes. While a few criticisms appeal to one as he reads, they are upon minor points and serve to emphasize rather than otherwise the value of the report.

It is suggested that certain terms be left undefined. Few teachers will dissent in regard to such words as *Whole, Part, Between, In, Through, Divide,* and this class of words not usually held to be technical terms of geometry; but there will be more or less difference of opinion upon the words *Point, Line, Surface, Plane,* and *Straight,* named by the committee. There may not be agreement in the language of the definitions. Many may even think they can not be satisfactorily defined, but most will agree, and this I take to be the position of the committee, that a careful discussion of their properties and limitations is necessary to clear of error the crude notions of pupils, and to serve as a basis of authority for future work. (It may be wisdom, for this association at some future time after mature deliberation, to formulate definitions or at least a statement of limitations in form of a discussion of above terms as well as of a few others. I would suggest that *Angle* be added to the list.)

There is no doubt that there is positive need of a set of words which shall respectively express the different meanings of the term *Equal. Equal* and *Equivalent* do fairly well; *equal,* for equal in all respects, and *equal in magnitude* have been used with reasonable accuracy. "*Congruent*" suggested by the committee for the equality of identity, is an awkward word and so foreign to the language of every day life that I doubt its expediency. If used by teachers and text, it may be forced upon the pupil while

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*Read before the Mathematics Section of the Central Association of Science and Mathematics Teachers, St. Louis, Mo., Nov. 30, 1907. Printed copies of the Geometry Report can be obtained from the Secretary of the Section, Miss Mabel Sykes, by enclosing a stamp to her at 438 E. 57th St., Chicago, Ill. The paragraphs refer to the respective paragraphs in the report.

in recitation, and possibly in conversation during the study of the lessons, but will soon pass out of use and *equal* will take its place.

The use of *Line, Sect,* and *Ray* for the different notions that have centered in the word *Line,* can not be too strongly emphasized. The word *Line* has been made to do triple duty long enough. But I confess I can not see why, for mathematics involving the area of the circle, *Circumference* should be discarded and *"Circle"* be made to do double duty. Is it not taking a step backwards and towards a confusion of ideas?

I believe the abolition of the term *axiom* will meet with general approval, and *assumption* can well take its place. There is wisdom, too, in delaying the statement of an assumption until it is needed. Its force will then appeal to the judgment of the pupil and he will be more likely to remember it and seek its aid when again it is needed, In studying the list of assumptions suggested by the committee, one is reminded frequently of the Hilbert axioms, and looks vainly for many of the axioms of Euclid. Many of the new assumptions will be welcomed, but why discard the time honored "Equals subtracted from equals", "Equals multiplied by the same or equals", etc.? Many writers on geometry and teachers have been unconsciously assuming certain truths that should have been stated, and the list cited by the committee calls attention to many of them. But I doubt, however, if the average boy could be made to think it anything but a waste of time to refer to numbers 4 and 5. If No. 7 is accepted, and the meaning of *angle* and *straight angle* is undestood, No. 9 follows as a consequence and need not be assumed. No. 8 is easily demonstrated, if No. 9 and the method of superposition is granted. Again granting the need of No. 11, No. 14 should be proved by it and not assumed. When it has been established that two circles can be made to coincide if their radii are equal, and that the magnitude of an angle determines the position of one side when the position of the vertex and the other side is known, why is it any more difficult to establish No. 16, viz., that "central angles inter cept equal arcs," than to prove that "two triangles are equal when two sides and the included angles are respectively equal?" If proof by superposition be denied, the problem may indeed be a difficult one.

Terms that depend upon the demonstration of a given theorem should follow that theorem, as *Distance, Center of a Regular Polygon, Apothem, Polar Distance,* etc. But there are advantages

in studying all of a group of related definitions, "*em bloc,*" in that it gives the student a classified notion of a given subject. For instance I would prefer the triangle definitions together; the quadrilateral definitions together, etc. If scattered about as they need to be used, they may, by the order in which they occur be thrown out of their relation of dependence upon one another, viz., the relation of *genus* and *species,* and in any case, the pupil will have more difficulty in building up the subject in his mind in its true classification. The meaning of *definition,* viz., the name of the proximate genus of the concept with a statement of the difference between the species of the genus, will be harder to appreciate; in fact, he may wholly miss the distinction of genus and species if not presented together in a classification.

The use of figures is sadly abused. The report wisely calls attention to the matter. What teacher does not find his class as a whole making all their figures a copy of those in the book? The pupil depends upon the figures in the text and fails to see that he gains the truth from the figures rather than from the theorem. If he can be induced to study the theorem with the figures covered and construct from it his figure, finally using the one in the text as a check, the tendency decried by the committee will be largely remedied. The figures will not be exactly like those in the book studied, and this very fact if encouraged will engender a pride in independent work. Many texts do not define *Corresponding Angles, Alternate Interior Angles,* etc., but simply make a drawing and refer to the angles in the various positions by name. In such cases, it is true one must depend upon pictorial evidence to determine which are alternate interior angles formed by the opposite sides of a parallelogram and the diagonal. If a genuine definition has been given, this definition becomes legitimate authority. It is true "that a demonstration should aim to recognize and acknowledge all the sources upon which it depends;" but it is true, too, that in the beginning, if this is done, the pupil will be lost in the maze of technicalities. If the main authorities are given, he can appreciate the argument, and no violence is done in *his* mind to the rigor of the demonstration. As he grows in strength more and more, he can be made to appreciate all the sources of dependence which should be credited.

One of the most common fallacies of students in original work is the tendency to redundancies lamented by the committee. To illustrate; We have all heard again and again, "Bisect the ver-

tical angle of an isosceles triangle and extend to the middle of the base" or "draw it perpendicular to the base;" "Draw a perpendicular from the vertex of an isosceles triangle to the base at its middle point." "Draw a radius perpendicular to a chord at its middle point," etc.

That part of the report which appeals to me most strongly is the challenge of the treatment so common in texts of proportion. For, to prove the truths in the theory of proportion algebraically and then apply them to geometric magnitudes without noting the necessary limitations, is, in the words of the committee, "extremely unfortunate." The distinction between *Number* and *Quantity* should be emphasized and both clearly considered in the theory of proportion with the consequent limitations in use of *Quantity* noted. For instance, a proportion of quantities can not be taken by alternation unless the units of the terms of both ratios are the same, because the definition of ratio would be violated. If Ratio is to be defined algebraically then Quantity must be measured, which at once demands a full appreciation of unit and number or coefficient. If the algebraic definition of product is to be used, such expressions as AB need interpretation. The pupil should not use it regardless of meaning; i, e., whether A and B are both numbers, or one number and one quantity; or as sometimes happens in both text and school room, both seem to be quantities. The using of symbols with no appreciation of their meaning is, to say the least, a waste of time. It would assist the student of geometry if in his algebra and arithmetic, the function of coefficient and unit could be more fully developed. In the case of two factors, as $3\times$(times) 5, he should know that the multiplier is the number or coefficient and the multiplicand the unit; that if both were numbers, either may be taken for the unit, according to convenience. In the case of addition of, say, 3×5 and 3×7, the pupil should appreciate that the unit should be selected for brevity in adding; as 7 threes and 5 threes; or in multiplication, of say, $6\times3\times5$, it is a matter of convenience whether to say 6×3 fives or 6×5 threes. Could the pupils be trained to a fuller appreciation of the number unit, the quantity unit, and their relation to coefficients, the addition of products in geometry would not seem so foreign to them; as in finding the area of a regular polygon in ½ B(base) $\times$ A(alt.) + ½ B $\times$ A, etc. If A and B have been taken as measures of lines, viz., numbers, he would see at once that the unit and coef-

ficient may be selected on the basis of what is demanded of the sum. In short, in his thinking, the pupil should be governed, as is any other workman, by the nature of his materials and what he wishes to make of them.

It is in the nature of the subject that each statement made in a demonstration should be based upon authority. But there should be a reason apparent if that authority is purely arbitrary. I doubt whether it is wise to assume a proposition that can be demonstrated, unless that demonstration is too difficult at the time for the pupil; for instance, the theorem of central angles and their arcs before referred to and other assumptions.

It is probably wise with some classes, or some pupils in a class, to assume those comparisons of ratios involving incommensurable numbers in the first course if two courses are scheduled, or entirely if the course is but a year. It is probably best in any case to assume, but clearly assume, those theorems concerning the multiplication and division of variables by constants, etc. To use these and other theorems in the process of demonstration without distinctly recognizing them as authority, has much to do with the confusion in the pupil's mind in regard to what constitutes a demonstration and his consequent loss of confidence in his power to reason. At one time, he is held sharply to account for authority that to his mind is puerile, and again he is taught to make statements that he can not understand, giving no authority. Is it any wonder that he is lost in the maze of it and resorts to any subterfuge to make his rank so as to be rid of the subject? The proposition: "Two variables are equal if their limits are equal," may well be assumed.

The position of the committee that "the more careful study of a much smaller number of propositions would achieve the same, if not better results" is, I believe, a too modest statement of the fact. I believe it would be nearer the whole truth to have said "would achieve far better results." It is to be deplored that the teachers feel constrained to drive every class and every pupil in the class over the same course. In consequence, they are impatient of the time required by pupils to really think out the new truths. The appreciation of a new concept is a slow process. The teacher's memory of the argument brings the result quickly and, judging too often from his own standpoint rather than the pupil's, he fails to give him sufficient time for real thinking. The pupil gets the mistaken notion that his slowness is more or less a

disgrace; hence, seeks other and questionable means to meet the requirements—the requirements too often simply a glib rehearsal of the language of the book. Less matter worked out to a full understanding,—the more independently the better, on the other hand, will give that confidence and pleasure that always accompanies mastery of a subject. Because each class is composed of pupils of varying ability, an elastic course is desirable. This can be accomplished by putting as the necessary course of theorems the minimum that will furnish sufficient matter to meet the usual college requirements. The other theorms dispersed through the work as exercises will give free play for the swiftest student, and each can get the fullest development possible. A rigid course perhaps suited to the medium holds back and disgusts the brilliant student, while it is absolutely discouraging to the slowest. The exercises again had better be distributed throughout the work from day to day, letting each lesson be a study of both necessary theorems and exercises, rather than the study of a chapter of theorems, followed by several pages of exercises. Hence, I believe the committee has done wisely in laying out a "tentative list" of necessary theorems, to guide the teacher who may not have the time or confidence to make such a course by elimination for himself. It is to be lamented that so many texts seem to be a cyclopedia of geometric truths, rather than a text to guide the teacher in his function as an instructor. Probably no two people without a conference would agree upon a particular list, probably no one would from year to year agree with himself upon a definite list. But such a list as the committee proposes will be suggestive and give courage to many to modify the course to suit the needs of their classes.

Finally, I would say, to emphasize, if I can, the position of the committee, that the closer one can keep to the experience of the student in high school geometry, the more value will accrue. To keep the student interested, he must see it a condition actually in his environment. When a notion is fully in mind, to let the pupil make application of it in physics, mensuraton, etc., is true pedagogy. The pupil must image each notion at the first, ,or his time is wasted; when once imaged, the symbol may safely stand for the notion, if well guarded. Hence, if necessary, the notion must be gotten from some physical representation. But it should be remembered that it is better to have the notion built up in his mind purely from the statement of it than from the geometric

figure, or physical representation. To spend one's time in building forms with tooth picks, wires, etc., or making shaded pictures and studying them, is a waste of time except for the slow pupil who at first needs this simple device to image the required notion. Such, might be termed kindergarten thinking. A higher kind, symbolic thinking, should take its place as soon as the mind is ready for it. The rare mind which can organize its facts, making new combinations, new constructions, perhaps developing new theories, has reached a still higher development. Geometry with an elastic course rationally administered furnishes opportunity for development for each kind of mind that seeks its assistance.

There is no doubt that if this report, revised, elaborated, illustrated and endorsed by this body, can be put into the hands of every teacher of geometry within the bounds of this association, such an improvement can be inaugurated in the teaching of the subject that its usefulness in the curricula of our secondary schools shall be fully equal to the claims of its most ardent friends. I wish to endorse the work of the committee and trust that the results may be utilized to the fullest extent possible

DISCUSSION OF THE REPORT OF THE COMMITTEE ON ALGEBRA.[1]

By Florian Cajori,

Colorado College, Colorado Springs.

It is a pleasure to discuss a report with which one is in substantial agreement. The main features of the report are a diminution of the amount of formal algebra during the first year course of the high school, an increase in the amount of problem work for that year, emphasis, throughout the high school period, upon the correlation of algebra and arithmetic, algebra and geometry, algebra and physics. If judiciously and skillfully carried out, these recommendations ought to lead to improvements in our teaching. A boy once decided to keep chickens, and to supply the family with fresh eggs. But the bantam chickens which he bought laid eggs that were miserably small. So the boy finally resorted to the expedient of hanging up in the chicken coop a big ostrich egg, with the notice. "Keep your eye on this and do your best." We teachers should keep our eye on this report and do our best.

[1] Read before the Mathematics Section of the Central Association of Science and Mathematics Teachers, St. Louis, Mo., November 29, 1907.

I said that good results would follow, if the recommendations are carried out judiciously and skillfully. Education has suffered more from the absence of a judicial attitude than from any other cause. We are continually going from one extreme to the other. Years ago the use of the spelling book, oral spelling, and parsing were over-emphasized; then followed a complete revolution in which these matters were shamefully neglected. At the beginning of the nineteenth century comparatively little stress was laid upon arithmetic. To girls it was hardly ever taught. Then followed a period when abstract arithmetic with the solution of difficult problems acquired such an ascendency in New England that it took the influence of General Francis A. Walker to check the excesses of that movement. A few years ago Chicago teachers were seized with a keen appreciation of the importance of the concept of ratio in arithmetic. As a result several of the western states, including my state of Colorado, were afflicted for years with a method which saw nothing in arithmetic but ratio. In the early part of the nineteenth century the algebras used in this country—the algebras of Euler, Lacroix and Day—were deficient in the formal treatment of algebra. Pupils did not get enough drill in factoring and the manipulation of fractions, radicals and exponents. Since that time teachers have passed to the opposite extreme. Now comes the reaction to that movement of excessive formalism. In early courses in algebra the formal side is to receive less attention, problem working greater attention. Does this mean that we are about to swing back to the other extreme? Is the saying of Washington Irving to become again true—the saying that "knowledge and genius, of which we make such great parade, consist but in detecting the errors and absurdities of those who have gone before, and devising new errors and absurdities, to be detected by those who may come after us?" I trust that *some* of our devices may stand the test of time.

The formal side of algebra is indispensible; moreover it inculcates powers of observation, as well as habits of accuracy. As I understand it, it is not the purpose of this report to materially decrease the amount of formal algebra in the high school; the report calls for a rearrangement of subjects, so that part of the formal work is postponed to the later years, and more of problem work is introduced during the first year.

The ideas of correlation are all important. They are like steam, which furnishes power for many useful ends, but which, under

careless and incompetent management may cause disastrous explosions. Particularly dangerous is the correlation of algebra and physics. A high school teacher of good mathematical ability told me the other day that as a high school pupil she hated the physics problems in algebra, and that she never really understood them until she came to her college course in physics. The experience of this person is, I fear, the rule rather than the exception. The early introduction of the graph is a desirable innovation, provided we do not attempt too much. The report before us is very sane on that point.

In my judgment the improvements in teaching which we aim to secure will not result from the introduction of startling new ideas. There is little or nothing novel in the schemes here presented.

The correlation of different mathematical subjects is found in eighteenth century text-books. The idea is as old as the hills. What we need most at the present time is not novelty, but a *judicious blending* of the old. When a child begins to paint, he indulges in all sorts of wild color contrasts. His lines are dead, and everything is loud and coarse. Using the same box of colors, a trained artist will make different selections, use better judgment, secure more careful blending, and obtain more subdued and artistic effects. He will draw living lines and secure infinitely subtle shades. What we need in the mathematical class-room is the trained artist who knows how to avoid excesses, who will apply correlation and other ideas just as far as he finds the interest and capacity of his students permit, and no further.

The correlation of algebra and arithmetic is important, not only because algebraic principles can be grasped more readily when connected with those already known in arithmetic, but because it affords the pupil practice in arithmetical computation. Mathematicians are proverbially poor computers. The reason is that they hardly ever have occasion to compute. It is one thing to be accurate in manipulating literal expressions, and another thing to be accurate with numbers. A person may acquire skill in one, but not in the other. To become skilled in both, he must keep in training for both. While teaching algebra we should insist on speed and accuracy in numerical as well as literal work. Drill in numerical work can be had in testing the accuracy of algebraic work by the substitution of particular values for the letters, as recommended in the report.

There are two recommendations in the report which I am inclined to reject. They are that multiplication and division should be taught together; also that radicals and exponents should be taught together. I must remark, however, that I have never tried the course recommended by the committee. I should think that a better course would be to teach the elements of multiplication first. When division is taken up, the results should be checked by multiplication, while the more difficult examples on multiplication might be taken up later, and checked by division. In that way multiplication and division play into each other. But the teaching of multiplication and division *together*, from the start, strikes me as a violation of the good old Pestalozzian maxim that we should teach only one thing at a time. I should recommend a similar course for radicals and exponents.

MEASUREMENT.

By Dr. George Bruce Halsted,

Greeley, Colorado.

Says Dr. E. W. Hobson: "It is a very significant fact that the operation of counting, in connection with which numbers, integral and fractional, have their origin, is the one and only absolutely exact operation of a mathematical character which we are able to undertake upon the objects which we perceive. On the other hand, all operations of the nature of measurement which we can perform in connection with the objects of perception contain an essential element of inexactness. The theory of exact measurement in the domain of the ideal objects of abstract geometry is not immediately derivable from intuition."

Arithmetic is a fundamental engine for our creative construction of the world in the interests of our dominance over it. The world so conceived bends to our will and purpose most completely. No rival construct now exists. There is no rival way of looking at the world's discrete constituents. One of the most far-reaching achievements of constructive human thinking is the arithmetization of that world handed down to us by the thinking of our animal predecessors.

In regard to an aggregate of things why do we care to inquire "how many?" Why do we count an assemblage of things? Why not be satisfied to look upon it as an animal would? How does the cardinal number of it help?

First of all it serves the various uses of identification. Then the inexhaustible wealth of properties individual and conjoined of exact science is through number assimilated and attached to the studied set, and its numeric potential revealed. Mathematical knowledge is made applicable and its transmission possible. Thus the number is basal for effective domination of the world social as well as natural.

Number arises from a creative act whose aim and purpose is to differentiate and dominate more perfectly than do animals the perceived material, primarily when perceived as made of individuals. Not merely must the material be made of individuals, but primarily it must be made of individuals in a way amenable to treatment of this particular kind by our finite powers. Powers which suffice to make specific a clutch of eggs, say a dozen, may be transcended by the stars in the sky. Number is the outcome of an aggressive operation of mind in making and distinguishing certain multiplex objects, certain manifolds. We substitute for the things of nature the things born of man's mind and more obedient, more docile. They, responsive to our needs, give us the result we are after, while economizing our output of effort, our life. The number series, the ordered denumerable discrete infinity is the prolific source of arithmetic progress. Who attempts to visualize ninety as a group of objects? It is nine tens. Then the fingers tell us what it is, no graphic group visualization. First comes the creation of artificial individuals, having numeric quality. The cardinal number of a group is a selective representation of it which takes or pictures only one quality of the group but takes that all at once. This selective picture process only applies primarily to those particular artificial wholes which may be called discrete aggregates. But these are of inestimable importance, for human life.

The overwhelming advantages of the number picture led after centuries to a human invention as clearly a device of man for himself as the telephone. This was a device for making a primative individual thinkable as a recognizable and recoverable artificial individual of the kind having the numeric quality, having a number picture. This is the recondite device called measurement. Measurement is an artifice for making a primitive individual conceivable as an artificial individual of the group kind with previously known elements, conventionally fixed elements, and so having a significant number-picture by which

knowledge of it may be transmitted, to anyone knowing the conventionally chosen standard unit, in terms of this previously known standard unit and an ascertained number.

From the number and the standard unit for measure the measured thing can be approximately reproduced and so known and recovered. No knowledge of the thing measured must be requisite for knowledge of the standard unit for the measurement. This standard unit of measure must have been familiar from previous direct perception. So the picturing of an individual as three thirds of itself is not measurement.

All measurement is essentially inexact. No exact measurement is ever possible. Counting is essentially prior to measuring. The savage making the first faltering steps, furnished number, an indispensible prerequisite for measurement, long ages before measurement was ever thought of. The primitive function of number was to serve the purpose of identification. Counting, consiting in associating with each primitive individual in an artificial individual a distinct primitive individual in a familiar artificial individual, is thus itself essentially the identification, by a one-to-one correspondence, of an unfamiliar with a familiar thing. Thus primitive counting decides which of the familiar groups of fingers is to have its numeric quality attached to the group counted. To attempt to found the notion of number upon measurement is a complete blunder. No measurement can be made exact, while number is perfectly exact.

Counting implies first a known ordinal series or a known series of groups; secondly an unfamiliar group; thirdly the identification of the unfamiliar group by its one-to-one correspondence with a familiar group of the known series. Absolutely no idea of measurement, of standard unit of measure, of value is necessarily involved or indeed ordinarily used in counting. We count when we wish to find out whether the same group of horses has been driven back at night that was taken out in the morning. Here counting is a process of identification, not connected fundamentally with any idea of a standard measurement-unit-of-reference, or any idea of some value to be ascertained. We may say with perfect certainty that there is no implicit presence of the measurement idea in primitive number.

The number system is not in any way based upon geometric congruence or measurement o fany sort or kind.

The numerical measurement of an extensive quantity consists in approximately making of it, by the use of a well-known exten-

sive quantity used as a standard unit, a collection of approximately equal, quantitatively equal, quantities, and then counting these approximately equal quantities. The single extensive quantity is said to be numerically measured in terms of the convened standard quantitative extensive unit.

For measurement, assumptions are necessary which are not needed for counting or number. Spatial measurement depends upon the assumption that there is available a standard body which may be transferred from place to place without undergoing any other change. Therein lies not only an assumption about the nature of space but also about the nature of space-occupying bodies. Kindred assumptions are necessary for the measurement of time and of mass.

Now in reality none of these assumptions requisite for measurement are exactly fulfilled. How fortunate then that number involves no measurement idea. But still other assumptions are made in measurement. After this device for making counting apply to something all in one piece has marked off the parts which are to be assumed as each equal to the standard, their order is unessential to their cardinal number. But it is also assumed that such pieces may be marked out beginning anywhere, then again anywhere in what remains, without affecting the final remainder or the whole count. Moreover measurement, even the very simplest, must face at once incommensurability. Whatever you take as standard for length, neither it nor any part of it is exactly contained in the diagonal of the square on it. This is proven. But the great probabilities are that your standard is not exactly contained in anything you may wish to measure. There is a remainder large or small, perceptible or imperceptible. Measurement then can only be a way of pretending that a thing is a discrete aggregate of parts equal to the standard or an aliquot part of it. We must neglect the remainder. If we do it unconsciously, so much the worse for us.

No way has been discovered of describing an object exactly by counting and words and a standard. Any man can count exactly. No man can measure exactly.

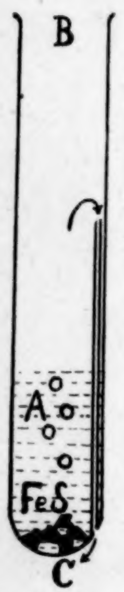
Arithmetic applies to our representation of the world, to the constructed phenomena the mind has created to help, to explain its own perceptions. This representation of things lends itself to the application of arithmetic. Arithmetic is a most powerful instrument for that ordering and simplification of perception which is fundamental for dominance over so-called nature.

Measurement may be analyzed into three primary procedures: (1) The conventional acceptance or determination of a standard object, the unit of measure. (2) The breaking up of the object to be measured into pieces each congruent to the standard object. (3) The counting of these pieces.

*A SIMPLE GAS GENERATOR FOR ANALYTICAL OPERATIONS.

JAMES MCCONNELL SANDERS.

The simple appliance described below has been found to be a convenient and practical substitute for the so-called "constant supply" apparatus of the Kipp type. It consists of a glass tube open at one end, and having a fine, almost capillary, tube fused into the other end, as shown in the figure.



The apparatus is charged, for example, for the preparation of hydrogen sulphide by dropping one or two small fragments of ferrous sulphide into the tube so that they lodge in the annular space (A), a few drops of dilute sulphuric acid are then allowed to run into the tube, and the mouth (B) closed with the forefinger. The apparatus may then be introduced into a test tube or beaker and used as an agitator, the gas escaping by way of the fine interior tube and the orifice (C).

For quantitative work, larger apparatus is used, and the mouth closed with a cork. For the preparation of a gas which requires the application of a gentle heat, the apparatus is allowed to touch the bottom of the test tube while immersed in the liquid contents of the latter, and that is applied to the test tube itself.

After use, the generation of gas is instantly stopped by directing a stream of water from a wash bottle into the interior, the outside is washed in the same manner, and the apparatus is then ready for use in a subsequent operation.

The appliance has been in constant use in the Mexican customs laboratories for the last five months, and in no case has it been found that the small inner tube showed any tendency to become clogged with precipitation.

*Chemical News, Nov. 29, 1907.

THE B-K SOLAR CALCULATOR.*

By Frank L. Bryant.

Erasmus Hall High School, Brooklyn, N. Y.

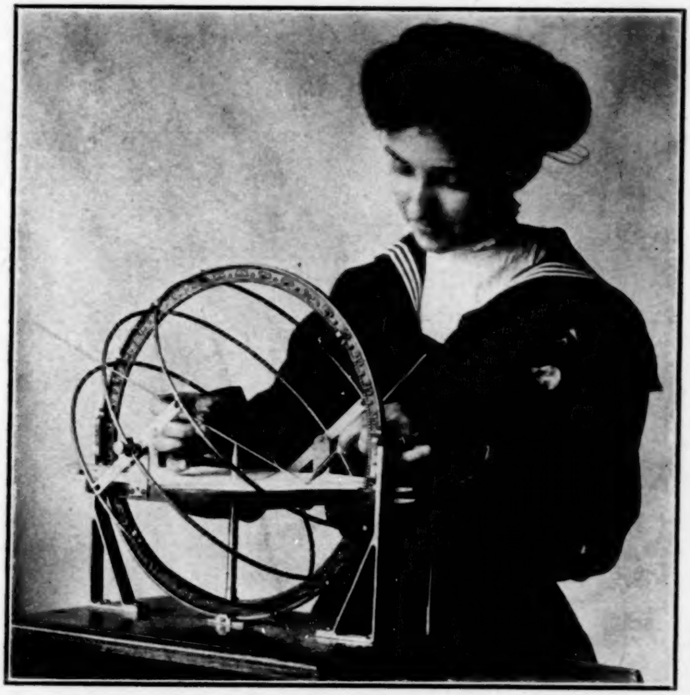


FIG. 1.

The changing form of shadow curves outlined by protruding points of opaque bodies due to the apparent motion of the sun at various times of year and in different latitudes may be of interest. The following curves were plotted by the use of a model which consists of a horizon disc surrounded by equinoxtial and solstitial sun-paths. The horizon disc, instead of being placed at the center of the sun-path circle, is lowered an amount equal to the height of a vertical post erected at its middle point. A sun-ray rod is used to locate the end of the shadow of the post by placing it in an hour mark notch on the sun-path and extending it over the post to the horizon disc.

*The B-K Solar Calculator is manufactured by L. E. Knott Apparatus Co., Boston, Mass. It is through the courtesy of this firm that we are here able to reproduce the drawings, they having kindly loaned us the plates.

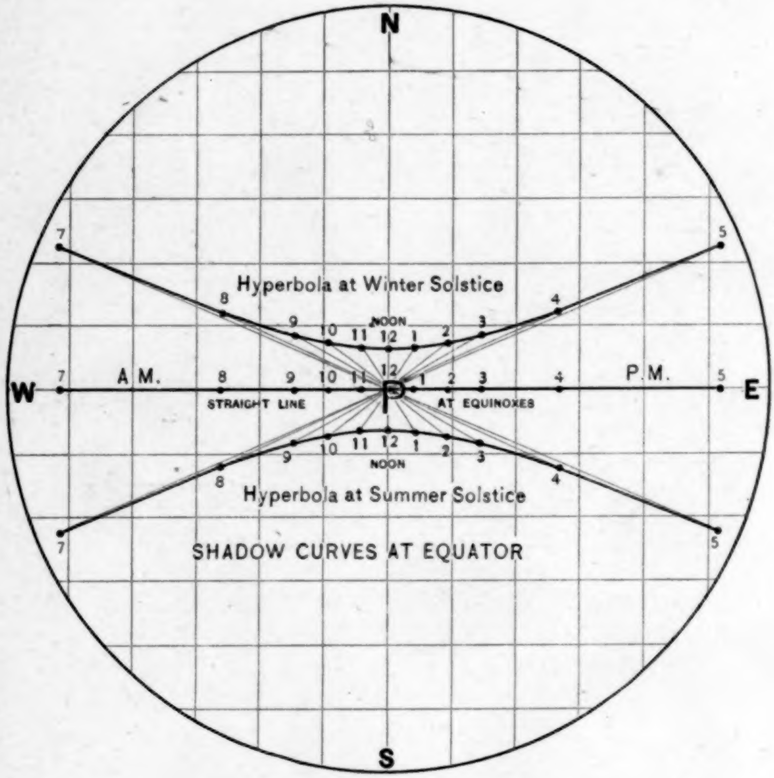


FIG. 2.

Points are thus located for each hour of the day and are then connected to form the curve. In Fig. 1 the student has adjusted the model for latitude 41° north, the sun-ray rod is placed in the eleven o'clock A. M. hour-mark on the sun-path for the winter solstice, and extends over the post to the horizon disc. A pencil is used to mark points thus located. The cardinal points are marked on the cross-section area bounded by a circle representing the horizon. Shadows extend from post, P, to shadow curve and are marked by the hour of the day at which they occur. The shadows in the morning at and up to about an hour after sunrise and also at and for about an hour before sunset are too slanting to fall on the horizon disc.

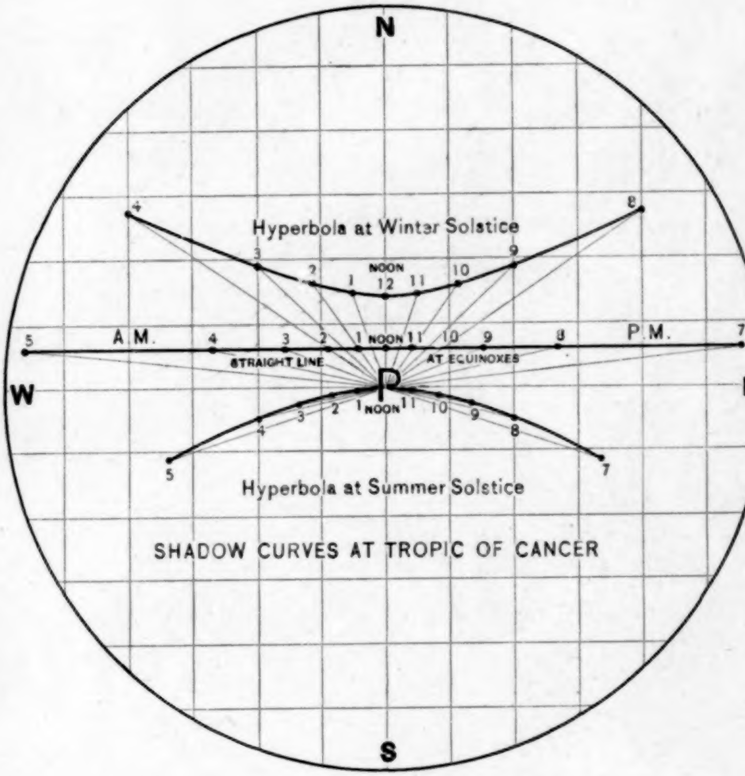


FIG. 3.

In passing from the equator toward the poles the sun-paths are inclined from the perpendicular an amount equal to the latitude of the observer. In Fig. 3 the curves have shifted northward from their position in Fig. 2, due to the inclining of the sun-paths 23½ degrees toward the south. At the solstices at all latitudes less than 66½ degrees the curves are hyperbolic. At the equinoxes a straight line is formed. The hyperbola formed at the summer solstice passes through the post, P, indicating vertical noon rays.

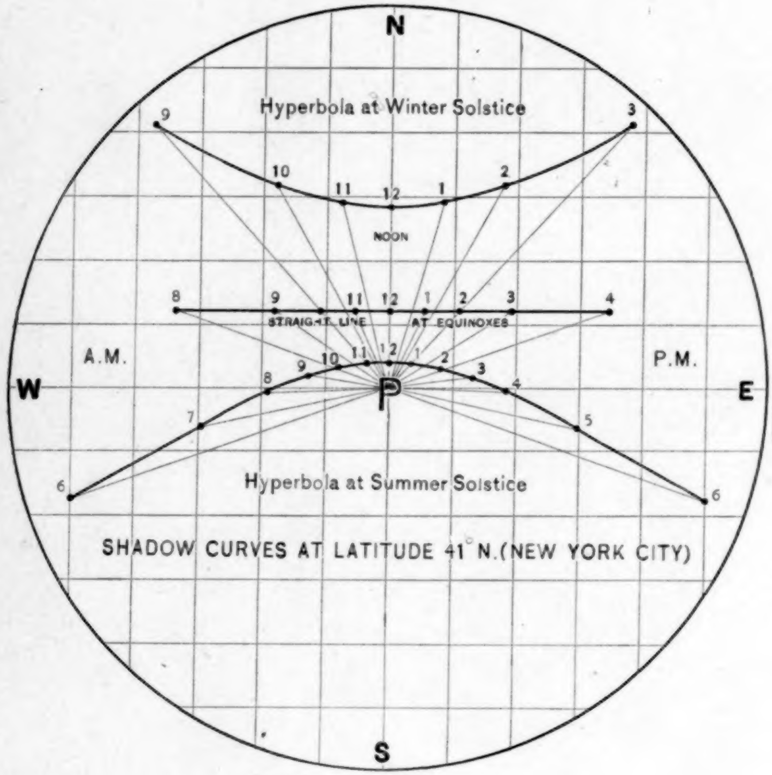


FIG. 4.

The curves shown in Fig. 4 are not easily predicted. The long winter shadows with long hour spaces may be compared with the short summer shadows with short hour spaces. The rapidly changing lengths of shadows in early morning and late afternoon is shown. By observing by means of the cross lines the direction of shadows for the summer solstice, Fig. 4, one may tell the time when the morning sun ceases to shine into north windows, also at what time the afternoon sun begins to shine into north windows. Problems of this kind may be suggested for other times of year and at different latitudes. The instrument as adjusted in Fig. 1 is in position to make curves in Fig. 4.

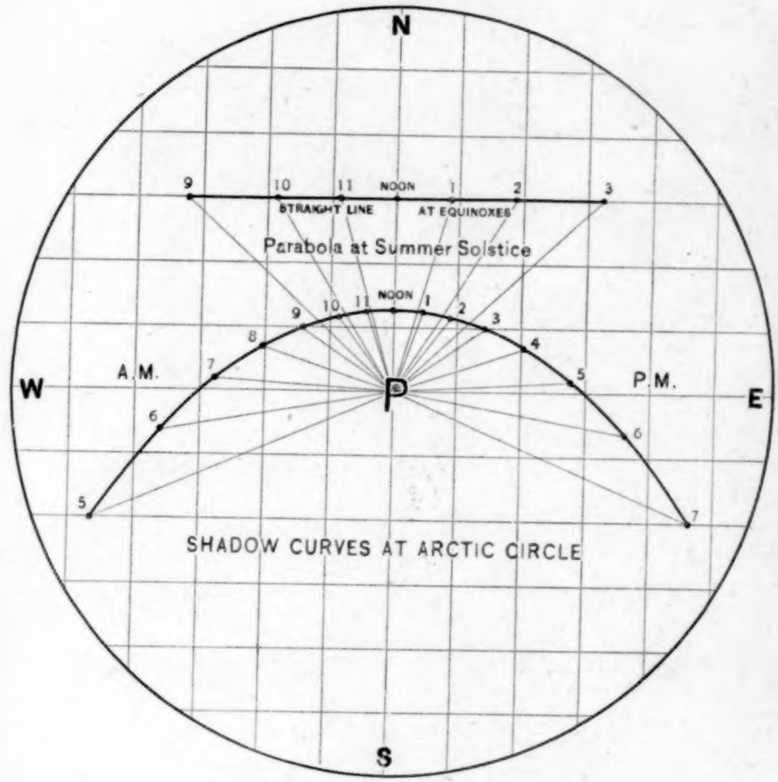


FIG. 5.

At latitude 66½ degrees north, Fig. 5, the southern limit of the "land of the midnight sun," the sun at midnight at the summer solstice dips down to the horizon at a point due north. Sunset and sunrise occur at this time and position. The midnight shadow of a post would be of infinite length and extend due south so that the parabola described during the 24 hours of the day cannot be completely represented on a paper disc 11½ inches in diameter.

The sun at the winter solstice just comes up to the horizon at noon at a point due south, so that a curve at this time of year is evidently not possible.

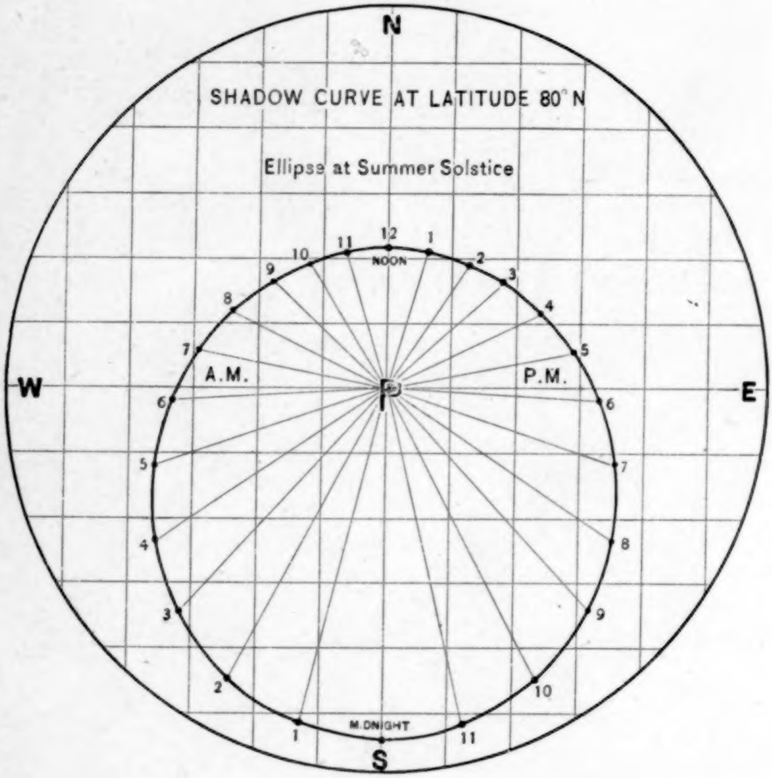


FIG. 6.

The latitude of 80 degrees north, Fig. 6, is the lowest latitude that would allow the shadow curve at the summer solstice to fall entirely on the paper disc. The form is that of the ellipse and shows the relative lengths of shadow for each of the 24 hours of the day. The noon sun at the equinoxes reaches a point only ten degrees above the horizon, but not enough to show the straight line always formed at this time. At the winter solstice at all latitudes above 66½ degrees north the sky sun-path runs below the horizon so that shadow curves are not formed.

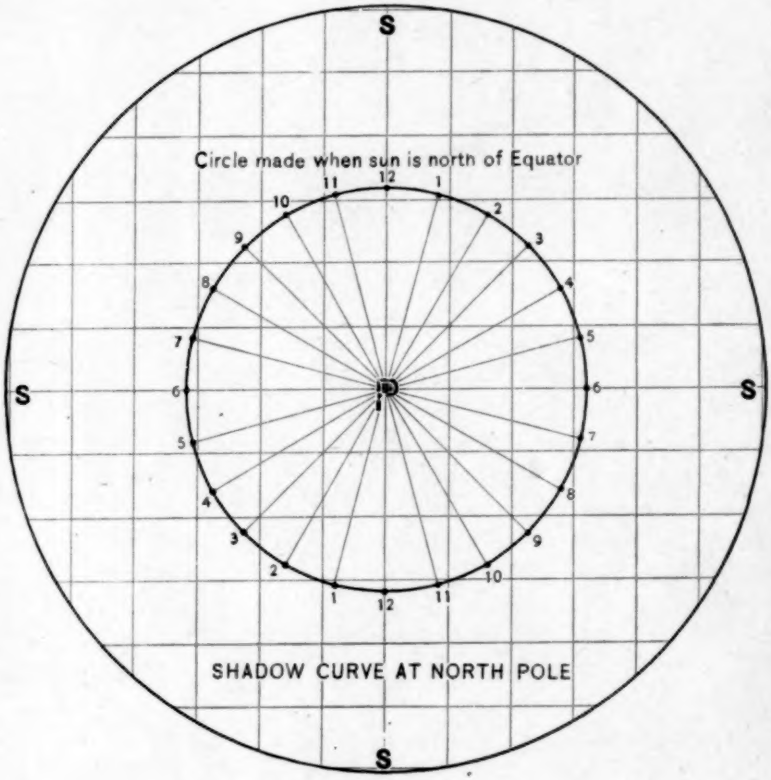


FIG. 7.

At the north pole, Fig. 7, when the sun is north of the equator the shadow curve is always a circle. When the sky path of the sun is highest the circle is smallest. When the sun is on the horizon a circle of infinite diameter would be formed.

In the construction of the model which makes the plotting of the curves possible, instrumental errors are reduced to a minimum. No attempt, however, was made to allow for errors due to dip of horizon, parallax, or refraction.

The form of curve that will be described at any latitude at any time of year may be summarized as follows:

1. Parabola, if latitude equal 90 degrees minus declination of sun.

2. Ellipse, if latitude is greater than 90 degrees minus declination of sun.

3. Hyperbola, if latitude is less than 90 degrees minus declination of sun.

If declination equals zero a straight line is formed. This is a special case of No. 3.

If latitude equals 90 degrees a circle is formed. This is a special case of No. 2.

The curve formed in latitude north is symmetrical with that formed in the same latitude south.

The only curve that can be formed at the same time in north and south latitude is a hyperbola.

The following laboratory exercises are helpful to students. The object is to clearly present problems occurring in our every-day lives that are difficult to master from a study of nature herself, without an expenditure of much time and extensive travel.

1. To determine the length of day at any latitude at different seasons of the year.

2. To find the change in direction of sunrise and sunset at different latitudes.

3. To plot shadow curves and interpret same as shown herewith.

Directions for these exercises are given in Handbook No. 26, published by the New York State Education Department, Albany, N. Y.

TO INSCRIBE A REGULAR POLYGON OF *n* SIDES IN A CIRCLE.

By Arthur J. Turner.

Montclair, N. J.

Let FE be the diameter, and B be the center of the circle. On FE construct the equilateral triangle FEA, and divide FE into *n* equal parts.

On FE take EC equal to two parts; connect A with C and produce to intersect the circumference at D. Let angle

DBE $=\theta$; then θ will be the angle at the center for a regular polygon of n sides.

Proof:

$$\triangle\ BCD = \triangle\ ABD - \triangle\ ABC$$

$$\triangle\ BCD = \tfrac{1}{2}\ BC \cdot BD \operatorname{Sin}\theta = \tfrac{1}{2}\ \frac{n-4}{n} \cdot R^2 \operatorname{Sin}\theta$$

$$\triangle\ ABD = \tfrac{1}{2}\ AB \cdot BD \operatorname{Sin}(90+\theta) = \tfrac{1}{2}\ \sqrt{3} \cdot R^2 \operatorname{Cos}\theta$$

$$\triangle\ ABC = \tfrac{1}{2}\ AB \cdot BC = \tfrac{1}{2}\ \sqrt{3}\ R^2 \cdot \frac{n-4}{n}$$

$$\tfrac{1}{2}\ \frac{n-4}{n} \cdot R^2 \operatorname{Sin}\theta = \tfrac{1}{2}\ \sqrt{3}\ R^2 \operatorname{Cos}\theta - \tfrac{1}{2}\ \sqrt{3}\ R^2 \cdot \frac{n-4}{n}$$

$$\frac{n-4}{n} \operatorname{Sin}\theta = \sqrt{3} \operatorname{Cos}\theta - \frac{n-4}{n}\ \sqrt{3}$$

Let $\frac{n-4}{n} = a$

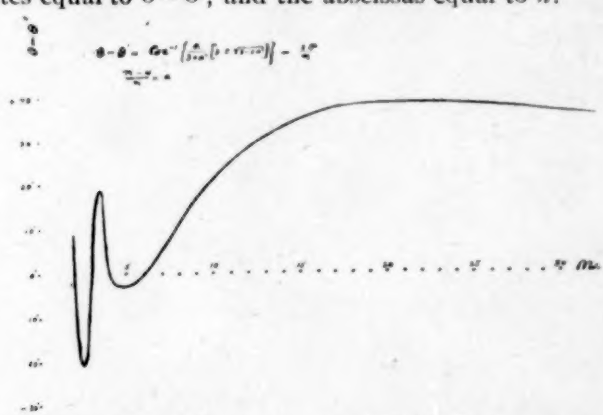
$$a \sin\theta = \sqrt{3}\ (\operatorname{Cos}\theta - a)$$

Solving for θ

$$\theta = \operatorname{Cos}^{-1}\left[\frac{a}{3+a^2}\left\{3 \pm \sqrt{3-2a^2}\right\}\right]$$

Now assign values to n and we obtain the corresponding values of θ. Call the true value of θ, $\theta' = \frac{360}{n}$. Then $(\theta - \theta')$ will be the error made in constructing a polygon by this method.

The accompanying curve has been constructed by making the ordinates equal to $\theta - \theta'$, and the abscissas equal to n.



The maximum value of $\theta - \theta'$, $(38' \ 18'')$ is reached when $n=23$, after which the curve approaches the axis of abscissas and $\theta - \theta' = 0$ when n is infinite. For four finite values of n $(n = 2, 3, 4, 6)$ $\theta - \theta' = 0$, and the construction gives us the regular triangle, the square, and the regular hexagon. For other integral values of n the error of closure of the polygon will be $n\ (\theta - \theta')$.

SOME VITAL POINTS IN THE TEACHING OF GEOMETRY.

By Herbert E. Hawkes,
Yale University.

We undoubtedly agree that at present the student is so crowded with book work that must be covered that all too little time is left for that most important work of all, a systematic, progressive, and careful training in the solution of original problems. Any relief that can be obtained from book work tends to greater efficiency there. Let us keep in mind that the explicit aim of the book work is the proof of the rules of mensuration of the circle and the polygons. The sooner and easier we can get to these proofs, consistently with sound reasoning, the better.

First, let us consider the relation of the axioms to the body of theorems in plane geometry. The dictionary says that axioms are self-evident—the text-book in geometry often says the same thing. But how about theorems? May they not be self-evident, too, even more clearly self-evident than the axioms? For instance, Euclid gives as his parallel axiom, "If two straight lines are cut by a third straight line, and the sum of the interior angles on the same side of the transversal is less than two right angles, then the two straight lines meet, and that on that side of the transversal where the sum of the angles is less than two right angles." He proves as a theorem that "the sum of two sides of a triangle is greater than the third side." Which statement I ask is more clearly self-evident? No one would say that Euclid's parallel axiom is more self-evident than many of his theorems. Self-evidence is, then, not the distinction between an axiom and a theorem. As a matter of fact, Euclid knew what a geometrical axiom is better than his followers for many hundred years. He realized, as we do to-day, that the geometrical axiom is a statement that we take as the basis for the logical development of our geometry, and that self-evidence has nothing to do with it.

We may assume many different sets of statements as axioms, and develop geometries from each of them. The geometries might differ in content to a wide extent, but would be in every case the logical consequences of their axioms. In recent years the attempt to find simpler axioms than those usually assumed

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From a paper read before the New York State Science Teachers Association, December 27, 1906.

has been very far-reaching. and we now know that a set of axioms for our geometry that comprises the fewest possible independent statements renders as theorems a large number of statements that we always tacitly assume in our textbooks, or else include in some more comprehensive axiom or definition. In order to push back the axioms to the fewest and simplest logically, proof must be given of theorems which would be included in a more extended set of axioms. In some of our common text-books this tendency is noticeable. We often see the theorems, "All straight angles are equal," "All right angles are equal," and the like proved. I ask, what is the use? Didn't we think these theorems were true before we proved them? Did the proof make us any surer of their validity? The students all say, "Why, any fool knows that!" True they have been given an opportunity to use their reason, but there are plenty of theorems left that are not self-evident in which they can get practice in logic. It strikes the student as too much like getting out of a battleship to kill a mosquito. The geometrical intuition of elementary students is outraged by such theorems, and they certainly encourage a lack of interest in and respect for geometry. Such theorems do not present any new facts, they do not teach the boy how to reason a whit better than the theorems that he considers worth proving, they certainly do not attune his mind to the verities of science, and they consume valuable time. The way out is very simple. So choose a body of axioms or assumed truths as to include all of those theorems that are self-evident. We know that we cannot use the most fundamental geometrical assumptions as axioms even if we want to, since it would make the geometry unteachable. Why, then, should we strain so strenuously in that direction? Make as axioms statements that anyone would accept as true in space without proof, and avoid those trivial theorems. This is not a matter that a teacher can be expected to carry out individually. It is one of the important points to keep in mind when selecting textbooks. In many schools a course in drawing or inventional geometry or some such subject precedes the study of geometry in which the pupil may become familiar with geometrical language, and through which these self-evident truths become indeed self-evident. This is of course a great help. I wish parenthetically to protest against the feeling that exists in some quarters that all the large colleges insist on this hairsplitting method of teaching geometry,

which is alike unscientific and unpedagogical.

I wish now to say a few words about the theory of limits. During ten years' experience in teaching geometry to students who, in general, have been carefully prepared by the best of teachers, I do not think I have met a dozen who understood the theory of limits. This is not the fault of the students, nor of the teachers, nor entirely of the textbooks. The difficulty lies in the fact that we are trying to do something that cannot be done. To give an average boy from 14 to 16 years of age a convincing sense that the proofs on limits are arithmetical in their essence. to train him to watch carefully the crucial point of the proof, namely, that the difference between the variable and its limit can be made less than any assigned value is a task so out of the swing of the rest of the geometry that it is doubtful if it can be done with any degree of success even with textbooks that are beyond reproach. You recall that each of the theorems in the first four books of geometry that use the theory of limits are divided into a commensurable and an incommensurable case. Now to begin with, the incommensurable case enters to meet a mathematical rather than a practical difficulty. No draftsman or mechanic was ever embarrassed because he could not measure a line exactly $\sqrt{2}$ feet long. The proof of the commensurable case is sufficient to give mental satisfaction to anyone in regard to the proof of any of those theorems. Even the fact that a regular inscribed or circumscribed polygon approaches the circle as a limit is evident after a few words of explanation.

Several ways are open to us. We may give enough time to the theory of limits so that the students understand thoroughly a rigorous treatment of the subject. This would require much more time than can be spared for it, and probably the plan could not be carried through in any length of time. We may continue to give the student the present proofs which they do not understand, and which are far from rigorous. If we recommend these proofs as rigorous, it is not honest; if we state that they merely serve to give mental satisfaction of the truth of the theorems, we surely fail, for the students saw the truth of the theorems clearly enough from the commensurable case. The theory of limits obscures rather than illumines the truth to their minds. We may explain the existence of the incommensurable case as a mathematical difficulty, illustrate it, and state that a rigorous treatment is out of the question at present, and that the proofs

will be found—somewhere else. In this way we save time, and we preserve the interest of the student and our reputation for scientific accuracy. Perhaps we shall decide to hear students flounder around over their heads in words a few years more before taking such a radical step. Perhaps the matter will not be settled in quite the way I have suggested. One thing, however, seems clear. From every point of view that I am able to take, the theory of limits in elementary teaching is a failure. If both secondary and college teachers suddenly discover that they agree on this point, the rest is simple.

FUNCTIONAL EXPONENTS.

By Arthur Latham Baker, Ph.D.,

Manual Training High School, Brooklyn.

"One of the most grotesque types of mathematical symbols is represented by $\sin^{-1}x$ for the angle whose sine is x. While $\sin^{\alpha}x$ means the α power of $\sin x$ for every value of α which differs from unity, it assumes an entirely different meaning for the special value of α."

I find the quotation above in School Science and Mathematics for May, 1907, p. 409.

The writer seems to have neglected the significance of *exponent,* a number showing how many times the operation of producing the functional expression is successively performed.

Thus if we denote by f the operation of squaring and adding one, we have, 1 being understood in the absence of an exponent,

$$f(x) = x^2+1$$
$$f^2(x) = (x^2+1)^2+1$$
$$f^3(x) = [(x^2+1)^2+1]^2+1$$

If f is a building up operation, then f^{-1} is the tearing down operation, the operation which exactly undoes the thing accomplished by f, so that

$$f^{-1}(f(x)) = x,$$

leading us back to exactly the point of starting.

Similarly

$$f^{-2}[f^3(x)]=f^{-2+3=1}(x) = x^2+1$$

The ordinary exponents of algebra are merely special cases of functional exponents; cases in which the functional operation

is confined to multiplication and division instead of being allowed to be any combination of the four algebraic operations, addition, subtraction, multiplication and division.

In the transcendental functions, e. g., sin x indicates the operation which builds up sin x from the arc x, viz,

$$\text{Sin } x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots\cdots$$

If we desire to indicate the tearing down operation, that which builds the arc from the sine, we must logically indicate it by $\sin^{-1}$

If we should write

$$x + \frac{x^3}{2\cdot 3} + \frac{1\cdot 3x^5}{2\cdot 4\cdot 5} + \frac{1\cdot 3\cdot 5x^7}{2\cdot 4\cdot 6\ 7} + \cdots\cdots$$

it would be difficult to recognize any connection with

$$\text{Sin } x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} - \frac{x^7}{\lfloor 7} + \cdots\cdots$$

But the moment we write

$$\text{Sin }^{-1}x = x + \frac{x^3}{2\text{-}3} + \frac{1\ 3x^5}{2\ 4\ 5} + \frac{1\text{-}3\text{-}5x^7}{2\text{-}4\text{-}6\text{-}7} + \cdots\cdots$$

the story is told, and the reciprocality of the two functional operations is stentor shouted.

To call this form grotesque because slovenly usage has given currency to a different and illogical interpretation is something like calling Addisonian diction grotesque because a majority of people use slang.

It must be remembered that the exponent can be in three different places, appended to the operand, to the functional operation, or to the result. Thus

$$\text{Sin }^{-1}\frac{\pi}{6} = 31^\circ 34'$$

$$\text{Sin }\left(\frac{\pi}{6}\right)^{-1} = \text{Sin } 1.91 = \text{Sin } 109^\circ\ 26' = 0.9430$$

$$\left(\text{Sin }\frac{\pi}{6}\right)^{-1} = 2$$

have entirely different meanings; as also

$$\text{Sin }^2\frac{\pi}{6} = \text{Sin }\left(\text{Sin }\frac{\pi}{6}\right) = \text{Sin } 0.5 = \text{Sin } 28^\circ 39' = 0.47942$$

$$\text{Sin }\left(\frac{\pi}{6}\right)^2 = \text{Sin } 0.27315 = \text{Sin } 15^\circ\ 39' = 0.26974$$

$$\left(\text{Sin }\frac{\pi}{6}\right)^2 = 0.25$$

The continental notation, arc sin x, has its value, but on the few occasions when we wish to use expressions such as

$$\mathrm{Sin}^{-2}\frac{\pi}{6}$$

it would be exceedingly clumsy. It is an accidental and sporadic notation which has no analogue in the case of

$$\log^{-1} 2 = 100$$
$$\log^{-2} 2 = 10^{100},$$

to say nothing of the higher functional forms.

gd and gd^{-1}
Sinh and Sinh^{-1}
Sn and Sn^{-1} etc.

Why is $\sin^{-1}$ any more grotesque than gd^{-1} or sn^{-1}?

To be exact, arc sin$x$ is a misnomer. It should be angle sin$x$.

In the case of the more familiar algebraic exponents, e. g., $x^{\frac{2}{3}}$, the $\frac{2}{3}$ indicates two multiplicative and three divisive operations, the first operation being the production of x from unity, the second the repetition of this operation, in conformity with the rule for multiplication.

The case of algebraic exponents is complicated somewhat by the fact that the symbol for the functional operation and for the result are the same, e. g.,

$$x(1) = x \quad , \quad x^2(1) = x^2,$$

where x indicates the functional operation of producing x from unity, in conformity with the definition of multiplication as the performing upon the operand of the operation which produced the multiplier from unity. The x indicates the functional operation or the result, according to the point of view.

Another complication arises from the fact that we have two ways of indicating the inverse functional operation, viz., by the negative exponent and by the position

$$x^{-2} = \frac{1}{x^2}$$

The ambiguity is not lessened by the fact that the inverse operation has received a special name of its own, division.

Instead of calling $\sin^{-1}$ grotesque, because slovenly usage has stolen its birthright, would it not be better to show loyalty to logical consistency and exactitude by insisting that those who mean $(\sin x)^{-1}$ shall use the proper notation therefor and leave $\sin^{-1} x$ in quiet possession of its hereditary and legal rights?

LAWS OF FALLING BODIES.

BY E. J. RENDTORFF,
Lake Forest Academy, Lake Forest, Ill.

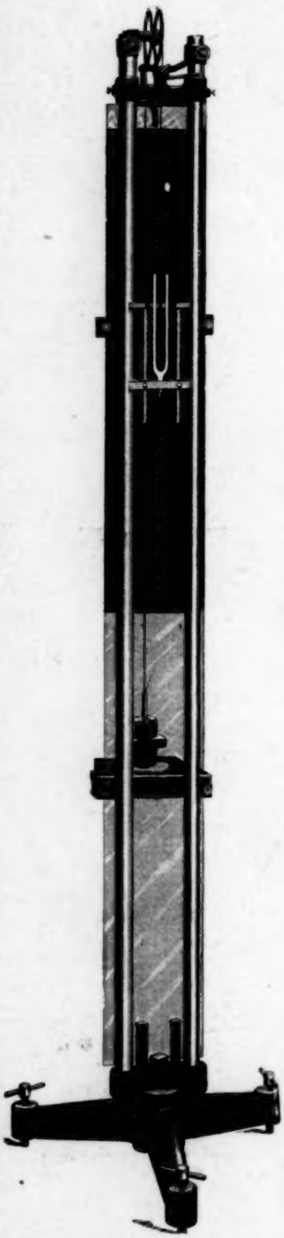


FIG. 1

I wish to submit an experimental determination of the laws of falling bodies which I believe has never been presented in any laboratory manual or teachers' journal

After having the students of the Lake Forest Academy perform the experiment for several years I consider it of sufficient value to be presented to the general public. The experimental errors are no larger than those of most work in mechanics, being usually less than 1%.

The apparatus employed is the falling body mechanism of Wm. Gaertner & Co., 5347-49 Lake Ave., Chicago, as shown in Fig. 1. It consists of a tuning fork of known rate of vibration, guided in its fall by two highly polished nickel plated steel rods. The prongs of the fork are slightly spread by means of an eccentric on the top of the frame. This eccentric can be drawn up into the frame by turning a lever, which releases the fork and sets it in vibration. A light metal stylus is attached to one of the prongs, and records the vibrations on a glass plate smoked with a flame of alcohol in which some gum of camphor has been dissolved.

The glass plate is held parallel to the guide rods and can easily be shifted sideways, so that a number of curves can be traced with one smoking of the plate. Two dash pots at the base of the instrument catch the falling frame, and take up the jar.

Adjust the instrument so that the guide bars are vertical, as indicated by an attachable pendulum. Remove the pendulum and place the smoked glass in position behind the fork. Tightly grasp the guide rods about 1 cm. below the falling frame and release the fork, withdrawing the fingers horizontally the moment the falling frame strikes them. Move the smoked glass several centimeters and obtain several other curves.

On striking the fingers the frame holding the fork comes to rest and then falls freely. Unless this is done, the release of the eccentric adds a component velocity not due to gravitation.

Remove the glass plate, place it on a table with the smoked face up, and select a curve indicating a fall from rest. Draw lines *a*, *b*, *c*, etc. Fig. 2 at the end of every fifth curve. Now place the glass plate in a trough with the smoked side down; place a meter stick on edge parallel to the curve. with one end at *a*, and determine the positions of *a*, *b*, *c*, etc. For each reading place the eye in such a position that the divisions on the scale and their images in the glass are continuous lines.

FIG. 2

Thus we obtain the successive spaces, *s*, fallen through in time, *t*. The time interval is not the second, but the time required for the fork to make five vibrations, and its actual value is immaterial.

The final velocity, *v*, at the end of any period of time, is the mean value of the difference between the total spaces fallen through at the end of the succeeding and the preceding periods, or—

$$V_t = \frac{S_{t+1} - S_{t+1}}{2} \qquad (1)$$

The acceleration, α for any time interval, is the difference between the final velocity of that period and the final velocity of the preceding period, or

$$\alpha_t = V_t - V_{t-1} \qquad (2)$$

From these equations the various final velocities and accelerations can be calculated.

Now tabulate the results in the form indicated below

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t	s	v	a	$\frac{v}{t}$	$\frac{s}{t^2}$	$\frac{v^2}{s}$	$\frac{2s}{t}$

Calculate the averages of columns (4), (5), (6), and (7).

From these averages it will be seen that, within the limits of experimental error

$$\frac{v}{t} = a \quad \text{or} \quad v = a\,t \tag{3}$$

$$\frac{s}{t^2} = \tfrac{1}{2}a \quad \text{or} \quad s = \tfrac{1}{2}a\,t^2 \tag{4}$$

$$\frac{v^2}{s} = 2a \quad \text{or} \quad v = \sqrt{2\,a\,s} \tag{5}$$

$$\frac{2s}{t} = v \quad \text{or} \quad s = \tfrac{1}{2}vt \tag{6}$$

The following table gives the work of one of my students of average standing. It was performed at the beginning of his elementary course and is typical of the results generally obtained with this experiment.

t	s	v	a	$\frac{v}{t}$	$\frac{s}{t^2}$	$\frac{v^2}{s}$	$\frac{2s}{t}$
1	.67cm	1.34cm	1.34cm	1.3400	.6700	2.6800	1.340
2	2.68	2.685	1.345	1.3425	.6700	2.6900	2.680
3	6.04	3.985	1.300	1.3283	.6710	2.6291	4.026
4	10.65	5.225	1.240	1.3062	.6656	2.5634	5.325
5	16.49	6.490	1.265	1.2980	.6596	2.5542	6.596
6	23.63	7.805	1.315	1.3008	.6569	2.5779	7.876
7	32.10	9.050	1.245	1.2928	.6551	2.5514	9.170
8	41.73	10.390	1.340	1.2987	.6520	2.5868	10.432
9	52.88	11.795	1.405	1.3105	.6528	2.6304	11.751
10	65.32	13.080	1.285	1.3080	.6532	2.6192	13.064
11	79.04	14.380	1.300	1.3073	.6532	2.6162	14.370
12	94.08						
		Average	1.3073	1.3123	.6599	2.6090	

$\dfrac{v/t}{a} = 1.0038$ or $v = a\,t$ Error = .8— of 1%

$\dfrac{s/t^2}{a} = .5041$ or $s = \frac{1}{2}a\,t^2$ Error = .8 of 1%

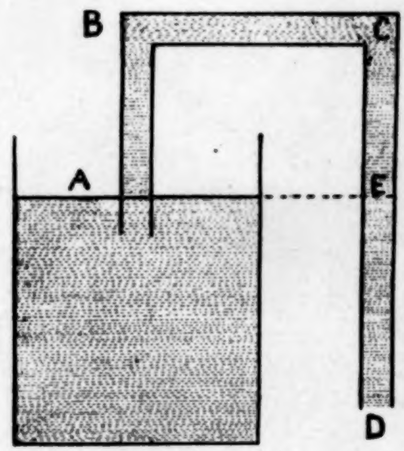
$\dfrac{v^2/s}{a} = 1.996$ or $v = \sqrt{2\,a\,s}$ Error = .2 of 1%

$\dfrac{2s}{t} = v$ or $s = \frac{1}{2}vt$ Error = .5— of 1%

AGAIN, "WHAT MAKES THE SIPHON WORK?"

By W. M. Bennett.
West High School, Rochester, N. Y.

The article in the January number of this magazine, on "What Makes the Siphon Work" attracted my attention. I do not think an average of seven out of fifteen texts are wrong in their explanation as the writer suggests. Nor do I think we should dismiss the matter by saying that it is clear that these statements given in a number of texts are incorrect.



The condemned statement is that the upward pressure in the short leg of the siphon at A, the level of the surface of the liquid, is equal to that of the atmosphere minus the pressure due to the column of liquid AB; also that the upward pressure at the bottom, D, of the long leg equals that of the atmosphere minus the pressure due to the column CD.

Consider the body of liquid in the tube. It is evident that there are *four* forces acting whose resultant determines the motion of this body, and they are the four forces mentioned above. These four forces may be combined mathematically in any order desired, but the gist of the matter is that atmospheric pressure at A and pressure of column CD both act in the same direction, toward the right, and their resultant is their sum. Also atmospheric pressure at D and pressure of column BA both act in one direction, toward the left, and their resultant is their sum. Now, if we combine these two resultants which act on the liquid body in opposite directions, we get as their resultant a force which *equals* the pressure at the column ED.

It is not obvious to the average high school student "That the downward pressure at D is greater than atmospheric by the pressure due to the column ED." And if it is not obvious, the only way it can be made so is by a full consideration of the *four forces* acting on the liquid body.

THE PLACE AND CONTENT OF A COURSE IN BIOLOGY IN THE HIGH SCHOOL.

By George William Hunter.

DeWitt Clinton High School, New York City.

It is a well known, although not always admitted fact, that there is in our American plan of education, a very considerable gap still existing between the grammar school and the high school. This hiatus, largely a difference in method, is due, to some extent at least, to a difference in the training of the teachers in the grades and the high schools, respectively. The former, usually a normal school graduate, with little or no training in the methods of research, sees in the pupil a plastic organism to be moulded to form the habits of correct thinking, according to certain fixed rules or laws of pedagogic method. The high school teacher on the other hand, fresh from the college or the graduate school, trained in the method of research, all too often tries with varying degrees of success, to impress these same methods in a more or less modified form upon the unfortunate victim. The "take it or leave it" dictum of the college cannot be at once administered in the first year of high school training. A transition from the last grammar grade to the high school will be natural only when each side accepts certain teachings in method from the other thus bringing the training into a natural sequence for the pupil.

Some factors accounting for the very great mortality in the early years of the high school have recently been discussed by Dr. A. M. Wolfson of the DeWitt Clinton High school in a paper as yet unpublished. He finds among the causes which take pupils from the high school in the city of New York are (1) lack of correlation in method with that of grades; (2) the unrest of adolescence which mitigates against application on the part of the pupil; (3) placing of child in the wrong kind of high school, giving of work unfitted to the mental makeup of the child; (4) economic reasons at home for the withdrawal of pupils.

The results of the factors just referred to are appallingly evident in the following figures obtained from the office of the City Superintendent of schools, New York city.

Pupils in High Schools in Classes I A (Beg'ng 1st year)	II A (Beg'ng 2d year)	III A (Beg'ng 3d year)	Percentage of Pupils leaving School by the end of 1st year	2d year
493	380	225	22.7%	54.4%
623	353	227	43.6%	63.6%
438	214	140	51.2%	68.1%
663	404	210	39.2%	68.4%
336	199	105	40.5%	68.8%
1,002	453	255	54.8%	74.6%
428	242	98	43.5%	77.8%
3,983	2,245	1,260	43.7%	68.4%

The above figures, with more or less variation, would be true of any large city in the United States. The results are considerably less marked in high schools in the smaller towns. Possible factors entering into this problem beside the ones already referred to, may be (a) larger and more crowded classes in the first year of the city high school; (b) the consequent lack of individual attention on the part of the teacher; (c) the placing of the stress on better teaching in the later years of the high school.

Science training has won its right to existence in the high school curriculum; the above statistics show that an extremely large proportion of our students go out to fight the battle of life with no more training that they get in the first year of the high school. What then, is the science which will best fit them, as future citizens, as future fathers and mothers, for their later obligations to society? If we bear the facts in mind, I do not see how we can escape from choosing as our first year science that of biology. Not only is it best adapted as the vehical of so called scientific method, but the subject matter is such as will be of most use to a coming generation of thinking men and women. As Hargitt[1] says, "It is one of the functions of biology to bring home to the consciousness of the youth that there is a unity and correlation of vital law common, not only to plant and animal, but to themselves as well."

A child, at the age of thirteen or fourteen, has the collecting instinct well developed, he is usually practical and is almost invariably interested in any manifestation of life. Especially at the age of beginning adolescence is there a newly awakened interest in generation and the development of the individual. This under-

[1] "The Place and Function of Biology in the Secondary Education." Proc. New York State Science Association. Bul. 28, New York State Education Dept., Oct. 1905.

current of thought, properly directed, is sufficient in itself to give the science of the biology of living things, culminating in the human organism, a lasting place as an introductory science in our high school curriculum. John Fiske's explanation of the tremendous importance of the period of prolonged human infancy in its relation to the future problems of the family and state may here be put to a practical use. The humanizing influence of such training here begun, will have a lasting influence on generations yet unborn.

To lead the child naturally into the channels of thought just hinted at, that "proper study of mankind," a modification of our present ideals in nature study training must be brought about. Such a modification is already taking place in some of our eastern cities. An ideal condition is pointed out by Prof. Hodge[2] in the following words: "High school biology .... would form the normal completion of elementary nature study and should aim to fit the great majority who do not study further for intelligent citizenship in harmony with the forces of living nature....I am constrained to conclude that the line between practical nature study and technical biology should be drawn between the high school and college rather than between the high school and the grades. And I take this position for two reasons. The first is that pupils of the high school age are not mature enough and lack the power of thought and the mental perspective to sense the significance of biological type.......The other reason is that there is a large body of biological knowledge, now neglected, which it seems to me would prove more valuable to the average citizen and the community than is the body of knowledge now represented by the the college course." This body of knowledge, represented by practical side of biological teaching, is to be developed by the teacher "so as to bring home to the pupil a realization of the dynamic forces of nature as ultimately expressed in the process of organic evolution."

If I interpret Prof. Hodge's paper aright I take it that he would place less insistance on the teaching of scientific method as such and develop to a greater extent the informational content of such a mass of biological knowledge. In this belief I heartily concur with Mr. Hodge. Experience of the past eight years has taught me that a child of fourteen is not interested in theoretic considerations, that he is eminently a practical animal and that he wishes to

[2]C. F. Hodge, Pedagogical Seminary, Sept. 1904.

apply his knowledge in terms of his environment. He takes the keenest interest in the life of an animal or plant in just so much as it touches his immediate or future welfare: above all, as I have already stated, he is keenly awake to problems of generation and heredity.

If utility is to be a consideration in the preparation of our future citizen, then this training could well begin in the grades below the present high school. Such is the nature of the experiment now being tried in the public schools of Boston. Sanitary science and hygiene has been so graded that the practical application of some of the questions of bacteriology, sanitary engineering and sanitation are now considered in some form by every child in the primary and secondary schools of Boston. And why should not elementary science teach the child why the town or city expends money for a department of street cleaning, in its sewers and system of water supply? Why should the youngest child not know why milk should be kept pure and how to do this; that meat and other food, if exposed to the air of a city street, may become the bearer of disease; or what infection and contagion means. The broader problem as applied to the regime of a city could then be applied in practice in the hygiene and sanitation of the school. Ultimately the effects of this training would be felt in the most important centre of all, the child's home.

It has been justly said that, "To know man and nature is the sum total of all human knowledge." With preliminary training along the lines indicated above our course in high school biology could have as its ultimate problem the place of man in nature, and his relation to the plant and animal world. Such a problem, necessarily making use of a physiological treatment of many types of plant and animal, lends itself well to the child's mind at this age, a time when extensive rather than intensive treatment of a subject is necessary. Rather a brief treatment of many forms and one principle driven home with each than a loss of coherency on the general plan and subsequent loss of interest on the part of the pupil. To again quote Prof. C. W. Hargitt: "Let us not overlook the ideals of biology, not the multiplicity of facts, not the large but slightly differentiated members of a group, but the relations of facts, significance of likenesses and differences, factors of adaptation and homology, fundamental processes, nutrition, growth, metabolism, reproduction, etc. Why not the morphology and physiology of living things, animals and plant conjointly and

comparatively rather than botany, zoology and physiology as distinct and more or less separate independent course."

The problems solved by biology are among the most important that man must meet. The constant struggle, the adaptations to meet new conditions, the problems of sex and heredity are constantly reappearing in one form or another to the average man. To be forewarned is half the battle; the best prepared survive. The laboratory and experimental study of plants and animals performed so as to illustrate certain of these principles and with a constant physiological treatment of the subject matter will be found to lend itself most admirably to this preparation for life and right living. Particularly in the treatment of plants and animals as living organisms can the subject of reproduction be approached and treated in a manner unobjectional even in mixed classes.

Perhaps the most practical bearing of a physiological treatment of plants and animals is found in the application of general principles of digestion, absorption, blood or sap manufacture, or metabolism, to the human mechanism. Here indeed is the ultimate test of the course in the first year, its adaptability to the real immediate and future needs of the child, the knowledge received to be by him disseminated, missionary like, to the tens of thousands of homes where ignorance still reigns. The culture value of biology is well known. It fosters liberality and openness of mind, appreciation of the great things in life, acquaintanceship with the mass of thought handed down by others. This culture value may be well obtained with young pupils by reference to the economic side of biology The informational content of economic biology, applied to the betterment of mankind, is most valuable. Moreover, in our cities at least, the spirit of commercialism, always present, is satisfied by the application of a science to the human good. And by this very sacrifice to the gods of the age the deeper meaning of biological teaching may often be introduced where it otherwise would never gain access because of prejudice of the uninformed parents. As a means to an end, the teaching of economic biology is justified, if for no other reasons. But other reasons vitally connected with the welfare of our nation, do exist. The need for care, preservation and in some cases replanting of our forests is appallingly shown in the recent reports of our national chief forester; the uses of the forest must be brought home before the necessity of its preservation is appreciated. And this is only one of the many factors which so few of the masses realize actually has to do with their

ultimate welfare, simply because they do not meet the conditions in their everyday life. Our national harvest, grains and fiber products, the enemies of the farmer and the fisherman, the value of selective planting and the part played by variation and other laws of heredity may each impress a lesson that will better prepare the future business man and voter to appreciate his place in the general plan.

The following outline will indicate sufficiently the general plan for a course in the first year of the high school. In this course five periods weekly, of from forty to forty-five minutes each, would be given. Three periods of laboratory or experimental work to two of recitation, demonstration or quiz seems to be a fair proportion. Such a course may best be introduced by means of a few simple chemical and physical experiments. These not only introduce the child to the method of science but also give to him some necessary concepts such as that of a chemical element, chemical compound, and an oxidation.

The botanical part of the course begins with the flower in the fall of the year. The flower in its adaptations for pollination, the interdependence of flower and insect, the gross structure of an insect here introduced to show the adaptations for pollination, the subject of fertilization and in connection with this, that most important topic of protoplasm and the cell.

The subject of fruits may be treated as briefly from the standpoint of ecology. Seed protection, seed distribution and the economic value of fruits are made the chief themes. With the seed and seedling experimental work is introduced to the pupil, definite experiments are set for him and the completed experiments are shown and explained by the pupil in the classroom. The work centers around the seed as a protection and source of food supply to the young plant. The factors influencing germination, the subject of foods and food tests and some elementary experiments to explain the meaning of digestion are here introduced. The morphological treatment of the root is only sufficient to demonstrate the nature of the fibrovascular core and the structure of the root hair. The physical changes and physiological bearing of osmosis are dwelt upon at considerable length; the economic importance of roots, directly as foods and indirectly their part in food manufacture is also treated here.

The work on the stem and bud is centered in the phenomena of growth and life activities, the reactions to light and other stimuli.

The function and, in a simple way, the structure of the stem with reference to its uses as a conductor and storehouse of food are touched upon. Economic treatment is given of the uses of stems as lumber and for food. An indication is given of the meaning of stem modification. The work of the leaf is emphasized, only enough time is given to structure to make clear the meaning of certain tissues in terms of the uses of these parts. Especial emphasis is placed on the leaf as a part of the living, working and breathing organism.

Cryptogamic botany is made use of for three purposes, (1) to open up the subject of systematic botany; (2) to illustrate life histories, especially in the matter of sexual reproduction; (3) to show the important relations existing between the lowest of plants and the highest of animals, as shown with the bacteria and fungi. Here an economic treatment of the relation has been found especially worth while. Throughout the botanical part of the course as well as the study of zoological types the ecological and natural history standpoint is used and constant effort made to reduce the use of scientific terms to a minimum, using only such terms as have application in a wider sense than those of mere classification.

In the purely zoological part of the course constant reference is made to physiological life processes each phylum is being used to illustrate some one process in particular. Inasmuch as the work with plants has ended at the lower end of the scale, the ascending evolutionary order may be used with the animal forms studied. It has been found, however, that the most satisfactory method is to take the frog (as the type of the vertebrates) in the spawning season and conclude the work in the early summer with the common insects abundant at that time.

Important economic relationships are pointed out in connection with each type studied; the relation of the protoza to disease, the commercial value of the sponge, division of labor and its meaning to society with the coelenterata, the starfish and its relation to the oyster fisheries, parasitism and regeneration with the worms. The crustaceans afford opportunity for reference to the work of the U. S. Fish Commission. The insects give an excellent field for elementary classification and life histories. Here as nowhere else does the economic relation apply. The mollusca are treated from the ecological and economic standpoint almost entirely.

The type used to demonstrate the vertebrate phylum is the frog, which serves to illustrate the morphology of man in connection

with human physiology. Human physiology is the culmination of the course in biology and should be so regarded by the child and teacher. When possible every comparison between plant and animal should be interpreted in terms of human physiology. The study of foods and digestion begun with the seedling and continued in the physiological treatment of the stem and leaf, is continued with the frog, it serving as an anatomical basis to illustrate the processes of digestion, absorption, blood manufacture and tissue building. Breathing, muscular activity, release of energy, and excretion may come later with the insects if the latter are used as laboratory types in the spring. The skeleton and the nervous system of man are far better appreciated with the frog used as an anatomical text. If the course for the student begins in the mid year it is better to begin with the seed and seedling, follow this with the root, stem and leaf, then take up such forms as are to be used as cryptogamic types and finish the botanical portion of the course with the flower and fruit when spring flowers are abundant. In the fall the treatment of zoological material begins with the insects, plentiful at this time, continues with the other phyla in descending the scale and culminates with the vertebrates, the frog again a type and used throughout to demonstrate human physiology.

For several reasons the course in biology as outlined for the first year of the high school is not well adapted for college entrance. The student is too immature to comprehend the philosophical teachings of biology and too inaccurate to make pure morphological work of value. The training in science method received in the first year of the high school puts the child in a receptive state for the later secondary school science courses. A type course in the third or fourth year of the high school to embrace a year in either botany or zoology given after such preliminary training as outlined above will give far larger values in science training than is now obtained and in addition will give college entrance training in biology which will be fully acceptable to college and university authorities.

PROBLEM DEPARTMENT.

IRA M. DELONG,

University of Colorado, Boulder, Colo.

Readers of the Magazine are invited to send solutions of the problems in this department and also to propose problems in which they are interested. Problems and solutions will be duly credited to their authors. Address all communications to Ira M. DeLong, Boulder, Colo.

Algebra.

87. *Proposed by J. Alexander Clarke, Philadelphia, Pa.*

If $\frac{a}{b}=\frac{c}{d}$ prove $\frac{bc+ca}{c-a}=\frac{2bcd+cd^2-ad^2}{cd-bc}$

I. *Solution by F. C. Mitchell, Danvers, Mass.*

By alternation and division

$$\frac{c}{c-a}=\frac{d}{d-b} \qquad \text{.......................................} (1)$$

Since $bc = ad$, $2bc = bc + ad$, and $2bc + cd = ad + bc + cd$, or, $bc + dc = 2bc + cd - ad$; whence

$$b+d=\frac{2bc+cd-ad}{c} \qquad \text{.......................................} (2)$$

Multiplying (1) by (2) we obtain the required result.

II. *Solution by H. E. Trefethen, Kent's Hill, Maine.*

$$\frac{a}{c}=\frac{b}{d}=\frac{bc}{cd},\quad \frac{a+c}{c-a}=\frac{bc+cd}{cd-bc}$$

$$\frac{ad+cd}{c-a}=\frac{d(bc+cd)}{cd-bc} \text{ and since } ad=bc$$

$$\frac{bc+cd}{c-a}=\frac{d(2bc+cd-ad)}{cd-bc}=\frac{2bcd+cd^2-ad^2}{cd-bc}$$

88. *Proposed by I. L. Winckler, Cleveland, Ohio.*

A man buying a rectangular garden whose perimeter is to be 100 yards, agreed to pay one dollar for every yard of the length and four dollars for every yard of the width. Required the least cost of a square rod of this ground.

Solution by W. L. Malone, Fern Hill, Wash.

Let x = the length in yards, then $50 - x$ = the width and $x + 4(50 - x)$ $= 200 - 3x$ = the entire cost. Cost per square yard, $y=\frac{200-3x}{x(50-x)}$ and therefore $x^2y - (50y + 3)x + 200 = 0$. The discriminant $(50y + 3)^2 - 800y$ $= 2500(y - \frac{9}{50})(y - \frac{1}{50})$ cannot be negative and y cannot have a value between $\frac{9}{50}$ and $\frac{1}{50}$. If $y = \frac{1}{50}$ then $x = 100$ and if $y < \frac{1}{50}$ x is imaginary. The least value of y is $\frac{9}{50}$ and the cost per square rod is $\frac{9}{50} \times 30\frac{1}{4}$ = \$5,445.

Geometry.

89. *Proposed by J. W. Ellison, Alcott, Colo.*

Construct a triangle having given the three medians.*

*[Note: This problem is given in Wentworth's Plane Geometry as Ex. 204, p. 132. D. L. Hines.]

I. *Solution by J. S. Brown, San Marcos, Texas, and J. Alexander Clarke, Philadelphia, Pa.*

With 2/3 of each of the three medians as sides, construct a triangle. Extend one of these sides its own length. Draw from the vertex opposite this side the median and extend this its own length. From the extended end of the first side draw lines to the extremities of the extended median. The resulting triangle is the one required.

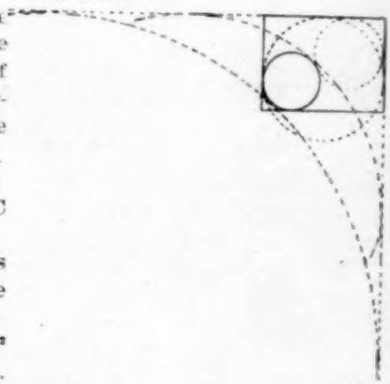
II. *Solution by H. C. Whittaker, Ph.D., Philadelphia, Pa.*

Construct a triangle with the three medians as sides. Then 4/3 of the medians of the latter triangle will be the sides of the required triangle.

90. *Proposed by H. E. Trefethen, Kent's Hill, Maine.*

A circle of radius 2 is inscribed touching two sides of a rectangle 6 by 8. What is the radius of another circle touching the other two sides and the given circle?

Solution by I. L. Winckler, Cleveland, Ohio.



First Method.—Let ABCD be the given rectangle, O the center of the given circle, and O′ the center of the circle tangent to the given circle and the other two sides of the rectangle. Through O draw OE $\perp$ BC, and through O′ draw GF $\perp$ OE, intersecting OE at F and DC at G. Draw OO′.

We have the following relations for determining the radius of the required circle,

$$(R+2)^2-(R-6)^2=(R-4)^2 \quad \text{.......................(1).}$$

$$(R-2)^2-(R-6)^2=(R-4)^2 \quad \text{.......................(2).}$$

From (1) we find R = 2.202 and 21.79; from (2) R = 4 and 12.

(1) The given circle and the circle whose radius is 2.202 are tangent externally and the second circle is tangent to the other two sides of the the rectangle.

(2) The given circle and the circle whose radius is 21.79 are tangent externally and the second circle is tangent to the other two sides of the rectangle, produced.

(3) The given circle and the circle whose radius is 4 are tangent internally and the second circle touches the other two sides of the rectangle.

(4) The given circle and the circle whose radius is 12 are tangent internally and the second circle touches the other two sides of the rectangle, produced.

Second Method.—Take the center of the given circle as origin and lines through it perpendicular to the length and width of the rectangle, respectively, as axes. The equation of the given circle is $x^2+y^2=4$ ....(1), and that of the required circle is, $(x+R-6)^2+(y+R-4)^2=R^2$,.....(2). We get from these.

$$x = \frac{R^2 - 20R + 56 - 2(4 - R)y}{2(6 - R)}$$

Substituting in (1) and reducing, we have a quadratic in y and the condition that the circle (2) is tangent to (1) is found to be $16(R^2 - 20R + 56)^2 (4 - R)^2 - 16[(4 - R)^2 + (6 - R)^2] [(R^2 - 20R + 56)^2 - 16(6 - R)^2] = 0$.* From this equation we find $R = 4, 12, 6, 6, 2.202+, 21.79+$. There are four circles whose radii are 4, 12, 2.202, 21.79.

Miscellaneous.

91. *Proposed by Ira M. DeLong, M.A., Boulder, Colo.*

Since the axes of an ellipse are at right angles the product of their slopes is -1; but since they are also conjugate diameters, the product of their slopes should be $\frac{-b^2}{a^2}$. Explain the anomaly.

Solution by E. L. Brown, M.A., Denver, Colo.

We have no right to say that the product of the slopes of two lines coinciding with the axes of an ellipse is -1. In general the product of the slopes of two perpendicular lines is -1; but when one line coincides with the x-axis and the other with the y-axis, their slopes are represented by 0 and ∞, and while $\frac{m - m'}{1 + mm'} = \infty$ it does not follow necessarily that $mm' = -1$, since in this case the numerator is not finite. As a rule in such a case the product of the slopes, 0 and ∞, really has no definite value.

Strictly speaking 0 and ∞ are not numbers at all, and cannot therefore operate upon a quantity or be operated upon, because all operations imply the existence of the quantities concerned. However, it is often possible and advantageous to put into such expressions conventional meanings when we have a *law* which governs the variables. For example, $\frac{x^2 - 4}{x - 2}$ literally has no meaning when $x = 2$, though there is an advantage in *assigning* it the value 4; similarly, $\frac{x^2 - 9}{x - 3}$ has no value for $x = 3$, though likewise there is an advantage in *assigning* it the value 6. Hence, *by definition* $\frac{0}{0} = 4$ in the first case but $= 6$ in the second case. $\frac{0}{0}$, *per se*, has no meaning, and we may assign it one only when there is a law governing the variables to guide us in making the assignment. It is important to remark that these definite values are not assigned in an arbitrary manner, but in one to conform to the laws which the variables in general obey.

Now let OA be the positive semi-major axis and OB the positive semi-minor axis of an ellipse; also let OC, OD, OE be semi-diameters, OC in the second quadrant, OD and OE in the first quadrant, and so that OD is perpendicular to OC, and OE conjugate to OC. Let m, m_1, m_2, be the slopes of OC, OD, OE respectively. Let the point C move

*[The double root 6, above, is extraneous, having been introduced by clearing of fractions after eliminating y. If we should eliminate x, the extraneous double root, similarly introduced, would be 4. Ed.]

toward B and at the same time D move so that OD remains perpendicular to OC. We have thus stated a *law* according to which these two lines vary, expressed by $mm_1 = -1$. When OC coincides with OB, OD will coincide with OA, m will equal ∞ and m_1 will equal 0; in this case it is reasonable to *assign* to $0 \cdot \infty$ the value -1. Now if the point E move so that OE and OC remain conjugate semi-diameters, again we have expressed a *law* according to which these lines vary, expressed in this case by the equation $mm_2 = -\frac{b^2}{a^2}$. Just as before, when OC coincides with OB, OE will coincide with OA, m will equal ∞ and m_2 will equal 0, but here it is reasonable to *assign* to $0 \cdot \infty$ the value $-\frac{b^2}{a^2}$, and there is no inconsistency in so doing. Hence the value of $0 \cdot \infty$ when 0 and ∞ are the slopes of lines that coincide with the axes of reference, should be defined so as to conform to the law which in general expresses a relation between these lines.

Hence *by definition* the product of the slopes of conjugate diameters when they coincide with the axes of reference, that is when they are perpendicular to each other, is $-\frac{b^2}{a^2}$, and not -1.

CREDIT FOR SOLUTIONS RECEIVED.

Trigonometry 76. H. E. Trefethen (two solutions).—2.

Geometry 80. H. E. Trefethen (two solutions).—2.

Algebra 82. Harold Blair, W. T. Brewer, G. E. Congdon, A. J. Lewis, F. C. Mitchell, Martha Shannon.—6.

Geometry 83. A. J. Lewis.—1.

Geometry 84. A. J. Lewis.—1.

Trigonometry 85. Harold Blair, G. E. Congdon, A. J. Lewis.—3.

Miscellaneous 86. A. J. Lewis. Also two incorrect solutions.—3.

Algebra 87. E. L. Brown, Walter L. Brown, G. E. Congdon, J. Alexander Clarke, Mary L. Constable, Iva Ernsberger, H. C. Feemster, A. M. Harding, R. P. Harker, Geo. W. Hartwell, O. S. Hoover, D. L. Hines, I. E. Kline, A. J. Lewis, W. L. Malone, F. C. Mitchell, John C. Reeder, A. W. Rich, Gertrude L. Roper, O. R. Sheldon, F. J. Taylor, Mary Clemmer Tracy, H. E. Trefethen, H. C. Whitaker, I. L. Winckler, G. B. M. Zerr.—26.

Algebra 88. E. L. Brown, Wm. B. Borgers, Iva Ernsberger, O. S. Hoover, W. L. Malone, A. W. Rich, H. E. Trefethen (two solutions), I. L. Winckler, G. B. M. Zerr. Also two incorrect solutions.—12.

Geometry 89. E. L. Brown, J. S. Brown, Walter L. Brown (two solutions), J. Alexander Clarke, C. S. Cory, Iva Ernsberger, H. C. Feemster, Geo. W. Hartwell, D. L. Hines, O. S. Hoover, A. J. Lewis, L. E. A. Ling, F. C. Mitchell, Ernest Parker, A. W. Rich, Wilfred Sherk, H. E. Trefethen, Mary Clemmer Tracy, H. C. Whitaker, I. L. Winckler, G. B. M. Zerr. Also one incorrect solution.—22.

Geometry 90. E. L. Brown, J. S. Brown, Walter L. Brown, G. E. Congdon, C. S. Cory, Iva Ernsberger, H. C. Feemster, R. P. Harker, Geo. W. Hartwell, D. L. Hines, O. S. Hoover, A. J. Lewis, L. E. A. Ling, Ernest Parker, A. W. Rich, F. R. Ritzman, O. R. Sheldon, Mary Clemmer Tracy, Th. Thorson, H. E. Trefethen, H. C. Whitaker, I. L. Winckler (two solutions), G. B. M. Zerr.—25.

Miscellaneous 91. E. L. Brown, Walter L. Brown, Iva Ernsberger, H. E. Trefethen, I. L. Winckler, G. B. M. Zerr.—6.

Total number of solutions, 109.

PROBLEMS FOR SOLUTION.

Algebra.

98. *Proposed by W. T. Brewer, Grand Rapids, Michigan.*

Between 6 and 7 o'clock, how far is the minute hand ahead of the hour hand when they are making equal angles with a line drawn from 10 to 4, both angles on the same side of the line?

99. Express $(x - a_1)\ (x - a_2) \ldots\ldots (x - a_n)$ as a determinant of the nth order with monomial elements.

Geometry.

100. *Proposed by E. L. Brown, M. A., Denver, Colo.*

Inscribe a square in a given quadrilateral.

101. *Proposed by H. E. Trefethen, Kent's Hill, Maine.*

The altitude of a triangle is 24, the bisector of the vertical angle is 25, and the bisector of the base is 40. Construct the triangle and compute the sides.

Miscellaneous.

102. *Proposed by I. L. Winckler, Cleveland, Ohio.*

Find the limit of $(\cos ax)^{\csc^2 bx}$ when $x = o$.

The next summer meeting of the American Mathematical Society will be held at the University of Illinois, Urbana, during September.

The Cambridge Botanical Supply Company by its new catalogue, No. 53, which has recently come to us, shows a very significant enlargement of its business. An unusually complete list of plant materials in desirable stages are offered in bulk. Prepared slides of plant and animal materials, lantern slides illustrating plants and animals, and their habitats; of industrial chemistry; of eminent men of science; together with the large list of laboratory and field apparatus will serve to make this company a still larger factor than it has been in providing science teachers with dependable materials needed in their work.

SCIENCE QUESTIONS.

By Franklin T. Jones,

University School, Cleveland, Ohio.

Propose questions for solution or discussion.
Send in solutions of questions asked.
Send examination papers in the sciences.

A liquid stands to a depth h in a vessel connected by a stop-cock with another vessel of equal volume and like shape standing at the same level.

The potential energy of the liquid is $\frac{mgh}{2}$

The stop-cock is opened so that the liquid stands to a depth h/2 in each vessel.

The potential energy is now $\frac{mgh}{4}$

What becomes of the 50 per cent of energy which has ceased to be potential? (Crew—Elements of Physics, page 107.)

"Chemistry has no place in the high school, but, if it must be taught, let it be mainly descriptive. * * * It is generally conceded that students who have had chemistry in the high school are the most unsatisfactory workers in college laboratories." (School Science and Mathematics, Feb. 1908, page 167.)

Is this criticism pedagogically correct?
Is it true?
Is it just, if true?

In the January, 1908, number of School Science and Mathematics the following question was asked:

"One gram, subject only to inertia, is acted upon by a force equal to one dyne until it has been moved one centimeter. Ten grams, subject only to inertia, is acted upon by a force equal to one dyne until it has been moved one centimeter. Is the work the same in each case, and if so, how?"

Solution by Chas. H. Korns, Bradford, Pa.

Answer: The work is the same in each case.

Energy is the capacity to do work, and is measured in the same units, namely, the erg, the joule, etc. When work is done upon any body, the work-energy is transformed into kinetic energy, and the kinetic energy is equivalent to the work done. Now the work done in each of the cases in the problem is one erg, and, as the following solution shows, the kinetic energy of the one gram mass and of the ten gram mass each is one erg, after each has been moved through a space of one centimeter by a force of one dyne.

[a] Acceleration (a)=1 cm. per sec. per sec. (by Definition.)

$S = \frac{1}{2}at,^2$
$1 = \frac{1}{2} \times 1 \times t^2,$
$t^2 = 2,$
$t = \sqrt{2}$
$v = at = 1 \times \sqrt{2} = \sqrt{2},$
$E_K = \frac{1}{2}\,mv^2 = \frac{1}{2} \times 1 \times (\sqrt{2})^2 = 1. \quad \therefore E_K = 1$ erg.
[b]. Acceleration (a) = $\frac{1}{10}$ *cm* per sec. per sec.
$S = \frac{1}{2}at^2,$
$1 = \frac{1}{2} \times \frac{1}{10} \times t^2,$
$t^2 = 20,$
$t = \sqrt{20} = 2\sqrt{5},$
$v = at = \frac{1}{10} \times (2\sqrt{5}) = \frac{1}{5}\sqrt{5},$
$E_K = \frac{1}{2}\,mv^2 = \frac{1}{2} \times 10 \times (\frac{1}{5}\sqrt{5})^2 = 1. \quad \therefore E_K = 1$ erg,

It is worth noting that the work done by a force is entirely independent of the mass of the body upon which it is acting. By definition, the work equals the force times the space through which the force acts.

Solution by Eugene C. Crittenden, Ithaca, N. Y.

By definition work is the product of acting force into distance moved, provided this distance is measured in the direction of the line of action of the force. Assuming that the one dyne of force acts in the direction of the one centimeter of distance moved, the amount of work done in each case mentioned in the problem is therefore one unit, that is, one erg. Notice that this does not in any way depend upon the mass moved.

This result is strictly in accord with the laws of motion and the principle of conservation of energy. It may be of interest to show this by a computation of the energy imparted to the two masses.

The acceleration given the one gram is one centimeter per second per second, that given the ten grams is one-tenth centimeter per second per second. The acceleration is uniform; hence the velocity acquired is given by

$$V = \sqrt{2 \times \text{acceleration} \times \text{distance}},$$

which gives for the one gram $\sqrt{2}$ cm. per sec., for the ten grams $\sqrt{0.2}$ cm. per sec. Substitution in the equation

$$E = 1/2\ mv^2$$

gives the kinetic energy for each mass as one erg, thus justifying our original conclusion that the work done is, in each case, one erg. Similar computations using any mass whatever will invariably give the same result; in other words, if the force and distance are given, the amount of work done is independent of the mass acted upon.

The principle of Conservation of Energy applied above ought to be thoroughly taught even in secondary schools. If its fundamental idea, namely, that Nature does not give us "something for nothing," were generally understood, men would be less eager to throw away their money in the development of Keeley motors, perpetual motion machines, capillary pumps, liquid air refrigeration, and other alluring pseudo-scientific enterprises.

Solution by W. M. Bennett, Rochester, N. Y.

The work done in both cases is the same, one erg. Work is the product of two factors only, force and space; hence to compute the work done in either of the given cases we do not need to know the mass, nor need the body be subject only to inertia. Nor do we care what time is required for the motion, nor what speed the object may have at the end of the journey of one centimeter. We do not care, either, how exhausted the agent may be who has been exerting that force and doing that work. The simple question is, has the doer been pushing with just that amount of push called one dyne while motion took place in the direction of that force over a distance of one centimeter.

If a man were asked to lay hold of a cord and pull just one pound on a vehicle while it traveled for one block, what difference would it make to him, except for possible æsthetic reasons, whether he were pulling on a coal wagon or a perambulator?

Again, we may look at the matter from another point of view. A very little experimenting under conditions approximating the cases given will make it evident that the two masses will traverse the one centimeter distance in quite different times, and, at the close of the journey, will have quite different velocities; so different in fact that $1/2\ mv^2$ will equal $1/2\ m'v'^2$. But this is only another way of stating that the work done in the two cases is the same.

Sometimes the student seeking clear notions in this matter who contemplates the thought of pushing with a force of one pound against a feather while it travels ten feet, begins to feel mental distress at the thought of the speed he would have to develop to chase the feather through that distance while persistently applying to it that pressure, rather strenuous—for a feather. But this comes from confusing two things, quantity of work and activity of an agent, or *work* and *power*. Obviously the statement of a problem in work is not at all concerned with the question as to whether some particular agent, or carrier of energy, is able to part with his energy—do the work—as fast as the conditions of the problem require, for that is not a question of how much *work*, but of how *fast*.

If you agree to pay one dollar to have a ton of coal carried up a flight of stairs, it is all one and the same, as far as amount of work is concerned, whether a small boy tugs at the job all day, or a coal heaver carries it up in an hour. It will have cost you one dollar for a fixed number of foot-pounds of work in any case.

EDIBLE FERNS.

An evergreen tree-fern in the Pacific Isles is a common article of food with the natives. The roots and the lower parts of the stem are soft and pulpy and have a pleasant smell and taste, so that the medulla of this fern, which abounds in a reddish glutinous juice, is nearly as good as sago. The silver tree-fern, a beautiful species from New Zealand, is said to be eaten in the same way.—*American Botanist.*

[Notes Dept. of Metrology.]

INCREASED COMMERCE WITH METRIC COUNTRIES.

An instructive table is the following, compiled by Mr. Frederick Brooks from the "World Almanac." It clearly sets forth the increasing needs that the U. S. Government should adopt the Metric System of weights and measures for its customs service. The table gives the percentage of commerce of the U. S.—exports and imports added together—for the last five years, and shows the rapid increase of trade with those countries whose customs service employs the Metric System. In each of the last five years more than half the foreign trade has been with such countries. In 1903 our trade was barely above 50 per cent; in 1907 it was almost 54 per cent, the other 50 and 46 per cent respectively being with those nations whose customs duties are reckoned in other systems. Can anything in commerce more clearly show the need of our Government's adopting the Metric System than a falling off of our commerce with non-metric and a proportionate increase with metric nations?

"Year ending 30 June, 1903,	50.3	per cent
1904,	51.5	"
1905,	51.4	"
1906,	53.1	"
1907,	53.9	"

"Our trade with Great Britain, which is conspicuous, consists in great part of our exports of materials, such as raw cotton and meat products, for which uniformity in weighing and measuring has not the same importance as for articles manufactured to sizes. A large proportion of it is in goods like breadstuffs and oils, measured by bushels and gallons in which there is a difference of magnitude between the United States and Great Britain." R. P. W.

PLANTS THAT SELDOM FRUIT.

The knowledge that the common white potato seldom produces fruit, is so widely diffused that the barrenness of the plant causes no comment. Indeed, since the tubers in a measure function as seeds we have partially transferred the name to them. It is usual to speak of potatoes intended for planting as "seed potatoes." Real potato seeds may be found, however, if one searches the potato-fields long enough, and from such seeds new strains of potatoes may be raised. The potato is not alone in its strange ways. Many other plants, of which the ground-nut and lily-of-the-valley are good examples, rarely produce seeds. It is noticeable that all such plants have other excellent and efficient means of propagation and it may be assumed that finding one method requiring less effort than the other they have gradually adopted it. When plants have more than one means of multiplying, as for instance, seeds above ground and tubers or runners below ground, they usually subserve two distinct uses: those below ground serving to multiply the plant in its own locality, and those above giving it a chance of gaining a foot-hold in distant lands.—*American Botanist.*

TO THE MEMBERS OF THE BIOLOGY SECTION, C. A. S. & M. T.

At the St. Louis meeting, the Chairman-elect of the Biology section was instructed to appoint a committee of three to formulate a biological creed—a statement of the fundamentals that should dominate in all biological instruction in secondary schools. This committee will consist of Otis W. Caldwell, Chicago, Chairman; T. W. Galloway, Decatur, Ill.; and H. W. Norris, Grinnell, Iowa. This committee will report next fall, and one meeting of the section will be devoted to the discussion of this topic.

Likewise in accordance with action taken at St. Louis, the other session of the section next fall will take the nature of a symposium on certain lines of coöperative field work to be known as "Calendar Studies." It is the aim of those interested to enlist as large a number as possible in a concerted attack upon the problem of out-door studies for biological classes, with the hope of obtaining some fairly definite results for presentation to the section.

The Executive Committee of the section will offer more detailed suggestions concerning this work in the next issue of SCHOOL SCIENCE AND MATHEMATICS. FRED L. CHARLES, Chairman.

NEW PROVISIONS FOR TEACHING PHYSIOLOGY AND HYGIENE.

At the recent annual meeting of the Illinois State Teachers Association, held in Springfield, Dec. 27-29, 1907, action was taken on the report of the committee appointed one year ago to confer with the W. C. T. U. relative to a new provision for the teaching of Physiology and Hygiene. The members of this committee are: Jas. E. Armstrong, Chicago, Chairman; George A. Coe, Evanston; A. F. Nightingale, Chicago; J. Stanley Brown, Joliet; C. C. Krauskopf, Chicago; and Fred L. Charles, DeKalb. Several meetings were held during the year, and while much progress was made, the committee from the W. C. T. U. were not empowered by that organization to concur in any new provision or any modification of the law, hence no joint action could be taken.

The report to the I. S. T. A. embodied the following provisions: (1) Compulsory instruction in hygiene (including the effects of alcohol and narcotics) in each grade of the elementary school, the emphasis being placed on the hygienic, moral, social, and economic rather than the scientific aspect of the subject; (2) a textbook course in human anatomy, physiology, and hygiene in seventh grade—(no text requirement in other grades); (3) a half-year course in high school, the place of this course in the curriculum to be determined by local authorities; (4) from time to time, the framing of a course of study in hygiene for the eight grades by a committee of the I. S. T. A., said course to be mandatory upon the schools of the state; (5) the preparation of a bill embodying these features, to be presented to the legislature at the earliest opportunity.

The committee's report was adopted by a vote of 163 to 16, and the committee was continued for one year with instructions to carry the proposed bill to the legislature. F. L. C.

GEOMETRICAL INCONGRUITIES.

G. W. GREENWOOD,
Dunbar, Pa.

The recommendation of the Committee on Geometry that the term "congruent" be used to denote that two figures have the property usually expressed by stating they are such that they could be made to coincide, is supported by a consideration of the contradictory and unintelligible use made of the word "equal" when employed in this sense.

Take any text in geometry which does not use the word "congruent," but which uses the word "equal" instead. We find as usual the "axioms": "If equals be added to equals, the sums are equal"; "If equals be multiplied by equals (equal numbers?), the products are equal"; etc. But as we are to apply the term "equal" only to figures having the same size *and shape*, these axioms are in general untrue; for if to two equal figures, in this sense, be added two other equal figures, respectively, the sums are in general *not* equal; they have the same size, but not necessarily the same shape. Again, suppose we have two equal parallelograms (not rectangles), ABCD and A′B′C′D′; draw the diagonals AC and B′D′. Then the triangle ABC is half the first figure and the triangle B′C′D′ is half the second, but as they have not the same shape they are not equal. That is, the halves of equals are not always equal. In like manner, by taking two figures of the same size but not of the same shape, that is, two unequal figures, we may sometimes add equals to them in such a manner that the sums may be equal; in other words, if equals be added to unequals, the sums are sometimes equal.

Not only is such a use of the term "equal" dangerous, but it is not in harmony with its use elsewhere. Outside of such texts as we have been considering, the term denotes either that two expressions represent the same number, or that two magnitudes have the same measure for some common property; for example, they may be equal in weight, in size, in kinetic energy, etc., while differing in all other respects. These same relations must repeatedly find expression in geometry also, as in figures having the same area but not the same shape; but having, in such texts as we are considering, unnecessarily and unwisely taken the term "equal" for a new use, we introduce the term "equivalent" and adopt for the study of equivalent figures a set of "axioms" similar to those stated, but which differ from them both in being unconscious and true. These considerations should convince us that the logical claims of such texts may be safely discounted.

If, however, we use the term "equal" to denote figures of the same length, area, or volume, no confusion will arise; it will be in complete harmony both with its antecedent general use and in such mathematical studies as algebra and arithmetic, where the student learns that one quart equals two pints. Similar criticisms apply to the use of the sign of equality.

To indicate the new and distinct idea of identity in size and shape we should likewise use a new term and a new symbol; for this purpose the term "congruent" and the symbol $\equiv$ or $\cong$ have been employed by leading writers, and the term being initially new it will not lead students—or teachers—astray on account of former associations.

PERSONALS.

During the last convocation week, Professor G. A. Miller, associate editor of this journal, was elected vice-president of the American Mathematical Society, Chairman of the Chicago Section of this Society, and Secretary of Section A of the American Association for the Advancement of Science. The last position is for a period of five years, while each of the other two is for one year only.

E. L. Hill, science master in the Collegiate Institute, Guelph, Canada, associate editor in biology on this journal, has accepted a similar position in the High School at Calgary, Alberta, Canada, at an increased salary.

In a letter to the editor, Mr. Hill says: "This province is very much alert in educational matters. The public school buildings are very fine. I have seen nothing so fine elsewhere in Canada as the public school buildings of Calgary. They are magnificent stone structures with modern equipment. The Board in Calgary is erecting a new High School with all modern appliances for science teaching at a cost of $75,000. This is not bad for a new town of 25,000 population.

"Strathcona, which is practically a suburb of Edmonton, the capital of the province, is erecting a similar school. Strathcona is to be the university town. There are a number of good positions likely to open up in the course of a year or two."

The Physics Club of New York has arranged with Professor Crocker, Professor Sever, and Mr. Arendt of the Department of Electrical Engineering, Columbia University, to give a course of demonstration lectures in electrical engineering so far as it bears upon high-school physics.

Abstracts of the lectures will be published in this Journal.

ANIMALS FROZEN TO ICE.

Ernest Thompson-Seton thus comments in the *Ottawa Naturalist* concerning a ruffed grouse, apparently in sound health, found with its tail feathers frozen into the ice crust, under a bush. "In the winter they commonly sleep on the ground, entering snowdrifts only in the coldest weather. It is absolutely certain that its tail could not have been frozen down, had there not been at the place some liquid. This may have been produced by a certain condition of the bird's bowels, or the sun's heat in such a sheltered spot may have melted the snow, so that it was wet when the bird went in, or finally, the bird's tail may have been wet when it went to bed, and a frosty night completed the dilemma.

"This you will remember is an accident of a class which happens every year to the foxes in Alaska. They sit down on the wet ice, thereby casting a shadow over it. In fifteen or twenty minutes the wet in the shadow has congealed, and the fox would be made prisoner but that he tears himself violently away, leaving much of his fur in the ice. The consequence is that in the spring of the year all the blue foxes have their buttocks more or less denuded of fur."

ARTICLES IN CURRENT MAGAZINES.

Astrophysical Journal for January: "Contribution to the Study of the Photosphere," S. Chevalier; "Observations of Saturn's Rings at their Disappearances in 1907, with a Suggested Explanation of the Phenomena Presented," E. E. Barnard.

Bird Lore for January-February: "The American Dipper in Colorado," Junius Henderson; "The Bird that Nests in the Snow," Sidney S. S. Stansell; "Redpoll Linnets," Lottie A. Lacey.

Forestry and Irrigation for January: "Report of Secretary of Agriculture on the Appalachian Watersheds"; "Southern Appalachian—White Mountain Forest Bill"; "How National Forests Would Affect the People"; "The Dominion of the Lone Cone," Olive M. McKinley; "Lumbermen's Views on Reforestation," C. H. Goetz. For February: "National Forests and Public Opinion," Lydia Adams-Williams; "Work of the Minnesota Forest School at Itasca Lake," E. G. Cheyney.

Journal of Geology for November-December: "The Witnalersrand Gold Region, Transvaal, South Africa, as seen in Recent Mining Developments," R. A. F. Penrose, Jr.; "The Glaciation of the Uinta Mountains," Wallace W. Atwood.

Manual Training Magazine for February: "The Influence of Graphic Art," Henry T. Bailey; "The Organization of Manual Training in the High School," Gilbert B. Morrison; "A College Course in Constructive Design," Charles R. Richards.

Mining Science for January 2: "Advances in Ore Dressing During 1907," J. M. McClave; "Some Experiences With Exploration Tunnels," A. Lakes; "The World's Increased Gold Production," F. L. Garrison; "Portland Cement in Rocky Mountain Region," Geo. J. Bancroft; "Diamonds—Their Occurrence and Methods of Mining," H. Leffman. January 23: "The Manufacture of Commercial Portland Cement," R. K. Meade; "The Dragoon, Arizona, Tungsten Deposits," R. W. Richards; "Mechanical Management of World's Gold Stock," Alex. Del Mar; "Mining Education in the State of Alabama," E. J. McCaustland.

Photo Era for January: "The Fourth American Photographic Salon," Wilfred A. French; "Hints About Photographic Chemicals," Phil M. Riley; "How to Color Photographs," Bertha I. Barrett; "Depth of Focus from the Standpoint of the Pictorialist," Dr. George H. Scheer.

Scientific American, Supplement for January 25: "The Stream of Planetoids," by Dr. Julius Franz. Supplement for February 1: "Fauna and Flora in Winter"; "The Mother Substance of Radium," Otto Hahn: "Sight in the Lower Animals," Professor R. Hesse. February 8: "Plant Breeding," Luther Burbank; "Theory of Freezing Mixtures," Chalkley Palmer.

The Auk for January: "A Long-Drawn-Out Migration: Its Causes and Consequences," Rev. G. Eifrig; "Notes on the Spring Migration (1907) at Ann Arbor, Mich.," Norman A. Wood; "Summer Birds of Southwestern Saskatchewan," A. C. Bent; "Bird Records from Great Slave Lake Region," Ernest Thompson Seton.

Barite, or barium sulphate, is used extensively in the manufacture of paint.

Antimony is usually purchased by smelters on a basis of 45 per cent of the metal.

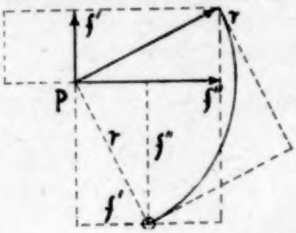
Aluminum is employed in Germany in the making of detonators or percussion caps.

Gold is mined in thirty-five counties of California and silver in thirty-three counties.

The melting point of pure metallic tungsten has been determined to lie between 2800° and 2850° C.

SOME NOTES.

Pythagoras's Theorem as a Special Case from the Law of Moments.



Let f′ and f″ be two forces perpendicular to each other, both acting at the same point. Their resultant is r. Construct the squares on f′, f″, and r. Taking 0 as the origin of moments, we get

$$f'f' + f''f'' = rr$$

i. e.

$$f'^2 + f''^2 = r^2.$$

Another "Proof" that I = 2.

I find it is a good exercise for the student to find the fallacy in the following proof.

The distance traveled by a body in t seconds when its acceleration is a, is given by the equation

$$s = 1/2.\ at^2.$$

Substitute for a the expression $v/t = s/t^2$, and it follows

$$s = 1/2.\ s/t^2.\ t^2.$$

From this we get $\quad 1 = 2.$

Query.

What happens if the length of the capillary tube is less than the capillary rise? Why?

H. Schapper,
University of Arkansas, Fayetteville.

A NOTE.

A Special Method to Multiply a Number Divisible by 7, by 43.

By Dwight S. Wiseman, No. 514 Robinson Building, Elmira, N. Y.

A special method relating to multiplying a number divisible by 7, by 43 is here given, and may prove of interest. Let us illustrate the method by an example.

Multiply 35 by 43.

We first divide the 35 by 7, which gives us the tens' and units' digits of our product. As the result of this operation gives only one digit, this must be put in units' place; and in the tens' place, we put a cipher, making 05.

To obtain the remaining left hand digit of our product, we must multiply the part of our product already found by 3, which gives 15; and our complete product will be 1505. This may be proved by the long method of multiplication.

When the first operation gives a quotient of more than two digits, the excess digits at the left, must be added to the same number of digits at the right of the result obtained by the second operation. Let us take an example or two.

Multiply 784 by 43.

(1) Dividing 784 by 7, we obtain 112.

(2) Multiplying 112 by 3, we have 336.

As, however, there are three digits in the first operation, the left hand digit 1, must be added to the right hand digit of the second operation, making 7; and our result will be 33712.

Of course, the addition can be done mentally; and only one line of figures need be set down in the product.

Multiply 7777 by 43.

$7777 \div 7 = 1111$; and $1111 \times 3 = 3333$. But as there are four digits in the result of the first operation, there are two digits at the left of that quotient to be added to two of the right hand digits of the product of the second operation. $11 + 33 = 44$; and the complete product is therefore 334411.

To multiply a number divisible by 7, by a number which is a multiple of 43, we take as many sevenths of the first number as the multiplier is multiple of 43. Thus for 43, we take one-seventh; for 86 we take two-sevenths; for 129, we take three-sevenths, and so on. For the remaining figures at the left, we multiply the result so obtained by the first operation by 3, as in multiplying by 43.

Multiply 63 by 86.

(1) 2/7 of $63 = 18$. (2) $18 \times 3 = 54$. Therefore the product is 5418.

(1) 3/7 of $77 = 33$. (2) $33 \times 3 = 99$. Product is 4933.

To multiply a number divisible by 7, by 143, 186, 243, 286, 343, we proceed in the first operation as in the case of 43, 86. But instead of multiplying by 3 for the second operation, we add 7 to 3 for each 100; thus for 143, 186, the multiplier would be $7 + 3 = 10$; for 243, 286, it would be $(7 \times 2) + 3 = 17$, and so on; for 343, 386, $(7 \times 3) + 3 = 24$.

Let us take an example. Multiply 84 by 243.

1st operation. $84 \div 7 = 12$.

2nd operation. Multiplier $(7 \times 2) + 3 = 17$. $12 \times 17 = 204$.

Therefore our result will be 20412.

Of course, when the number divisible by 7 is very large, and there is a hundreds' place in the multiplicand, the method will not be so advantageous as the multiplication of the quotient produced by the first operation by the correct multiplier to produce the remaining digits, may have to be written out.

[EDITORS' NOTE.—The *rationale* for Mr. Wiseman's algorithm for multiplying multiples of powers of 7 by 43 is not far to seek, and we leave its discovery as an exercise for the interested reader. If your explanations are sufficiently concise and complete, SCHOOL SCIENCE AND MATHEMATICS will publish one, and accredit others duly, on receipt of solutions.—THE MATHEMATICAL EDITORS.]

Large quantities of arsenic are now obtained as a by-product in smelting ores.

The present production of the U. S. Steel Corporation is about 25,000,000 tons annually.

AMERICAN NATURE-STUDY SOCIETY.

The American Nature-Study Society was organized at Chicago on January 2, 1908. Its purposes, as stated in the adopted constitution, are: (1) to promote critical investigation of all phases of nature-study (as distinguished from technical science) in schools, especially all studies of nature in elementary schools); and (2) to work for the establishment in schools of such nature-study as has been demonstrated valuable and practicable for elementary education.

Its membership consists of teachers and others who are interested in nature-study for schools and whose applications for membership have been approved by the Council. The annual membership fee is one dollar, payable before February 1, or upon election to membership in case of new members.

The Council for 1908 consists of the following officers: President, L. H. Bailey, Cornell University; Vice-Presidents, C. F. Hodge, Clark University; F. L. Stevens, N. C. College of Agriculture; V. L. Kellogg, Stanford University; W. Lochhead, Macdonald College, Quebec; F. L. Charles, DeKalb (Ill.) Normal School; Secretary-Treasurer, M. A. Bigelow, Teachers College, Columbia University; Directors (for two years), D. J. Crosby, U. S. Department of Agriculture; C. R. Mann, University of Chicago; S. Coulter, Purdue University; H. W. Fairbanks, Berkeley, Cal.; M. F. Guyer, University of Cincinnati; (for one year), O. W. Caldwell, University of Chicago; G. H. Trafton, Passaic, N. J.; F. L. Clements, University of Minnesota; Ruth Marshall, University of Nebraska; C. R. Downing, Marquette (Mich.) Normal School.

The constitution adopted provided for an official monthly journal to be published under the direction of the Council; and the well-established journal of nature-study, *The Nature-Study Review*, will be transferred to the Society. The annual subscription price ($1.00) of this journal is included in the membership fee of the Society ($1.00) provided that this fee is paid in advance; but subscribers to *The Review* are not enrolled as members of the Society unless elected after filing application. For the purpose of stimulating local interest, sections of the Society will be organized in various states and cities. Annual meetings will be held, usually in connection with the national scientific or educational societies. A directory of members will soon be published and revised annually.

The Chicago meeting was well attended by scientific men and dozens of others who were unable to be present expressed great interest in the movement. It is very important that at least one hundred fellows of the American Association for the Advancement of Science should be enrolled as members of the American Nature-Study Society. Their influence is greatly needed in the Nature-Study Society directly, and indirectly in establishing desirable relations with the American Association for the Advancement of Science.

For full information concerning the Society, or in sending in applications for membership, address M. A. Bigelow, Secretary, Teachers College, New York City.

REPORT OF MEETINGS OF THE PHYSICS AND CHEMISTRY SECTIONS, C. A. S. AND M. T., McKINLEY HIGH SCHOOL, ST. LOUIS, MO., NOV. 29 AND 30, '07.

Prof. H. N. Chute, chairman of the Physics section, being absent, Mr. G. A. Abbott of Indianapolis, chairman of the Chemistry section, presided over the joint sessions of these sections. The meetings were held in the Auditorium of the High School, Friday afternoon and Saturday morning.

After announcing the nominating committees, the Saturday afternoon trips, etc., the chairman introduced Prof. Wm. D. Henderson, Univ. of Michigan, who addressed the sections on "The Present Status of High School Physics." The following were mentioned as showing the present situation; The spirit of unrest—warm discussion and argument on many points, especially about the "disturbers of the peace," mathematics, and quantitative work—teachers are at sea as to what are points of the subject—courses in phenomenology, *i. e.*, easy physics. The speaker also pointed out a few things which will help and are helping solve these problems. There should be no interference between the High School and the University. Discussion from both points of view, coming from all parts of the country, shows the need of all working together. The "Syllabus" being developed is a result of this exchange of ideas. It will help to arrive at what is fundamental and to unify opinion on this as a working basis. The sections seemed to concur with the speaker that physics is not an easy subject and cannot be made such, and that "phenomenology" is a "kindergarten method" and as such should not replace present methods. It may supplement the present work where it gives useful information and adds interest. It is nonsense to try to make physics an easy study. Fundamental laws should be properly taught, qualitatively and quantitatively, well illustrated, some mathematics but not too much. Above all the teacher is urged to remember the pupil and to so adapt the work to his needs that it will increase his knowledge, broaden his experience, and develop his self control.

On account of the fullness of the program, little opportunity was given for discussion of any of the papers.

Prof. W. A. Noyes, Univ. of Illinois, then gave an interesting account of the "Work of the U. S. Bureau of Standards." He spoke especially of his work there, determining the combining weights of hydrogen and oxygen, and of the methods employed.

The last subject on the morning program was presented in a carefully prepared paper by Mr. Chester B. Curtis, Central High School, St. Louis, the topic being, "The Interrelation of the Physical Sciences in the High School Course." He showed in what respects each subject would correlate with the other if it precedes, and concluded that by all means Physics should precede Chemistry. The report of the committee of ten was referred to, as was also the minority report of the same committee. To determine to what extent this had influenced courses of study relative to these two subjects, Mr. Curtis sent out a questionaire, the data from which furnished some interesting facts on this subject. Physics was

shown to precede Chemistry in the third and fourth years respectively.

The first business of the Saturday morning session was the report of the committees on nominations. Those nominated for the coming year for the Physics section were: Chairman, F. E. Goodell, North Des Moines High School, Des Moines, Iowa; Vice-Chairman, Lynn B. McMullen, Shortridge High School, Indianapolis, Ind.; Secretary, Wm. M. Butler, Yeatman High School, St. Louis, Mo. For the Chemistry Section: Chairman, Fredus N. Peters, Central High School, Kansas City, Mo.; Vice-Chairman, Geo. C. Ashman, Bradley Polytechnic Institute, Peoria, Ill.; Secretary, John P. Drake, Western State Normal School, Macomb, Illinois. On motion they were unanimously elected.

The committees on the revision of the Physics Unit reported progress as follows: It has coöperated with similar committees elsewhere, has sent out three circular letters, and held a joint meeting in New York. The work of the committee is proving a strong influence in many quarters, but its final report cannot be made until the 1908 meeting. The report as filed was accepted and the committee given another year to continue its work. The committee is: Messrs. C. R. Mann, C. F. Adams, and Chas. H. Smith.

Prof. Lindly Pyle, Washington University, gave a very interesting and suggestive paper on "The Use of the Graphic Method in Determination of Physical Laws." The method described is one for laboratory instruction only, and can be used in part in high school work in teaching this subject and the meanings of the curves plotted. To show the character of the curve for $m \propto l$ in $m=k\ l$, the weights and lengths of several brass rods of varying lengths were plotted. The curve for the weights and lengths of metal squares showed the nature of the curve for $m \propto l^2$ in $m=k\ l^2$. Similarly the nature of the other characteristic curves and their equations were shown. The method certainly has several advantages: The data can be easily and quickly obtained, the relation between the variables is readily recognized in the objects measured, and the equation for this relation is easily stated. The paper was well illustrated with materials used, and curves and equations obtained therefrom.

Mr. L. B. McMullen, Shortridge High School, Indianapolis, discussed in a very pointed paper "Can Physics be Taught to High School Pupils as a Mathematical Science and at the Same Time be Made Interesting?" His answer was a strong affirmative. The ideas brought out in the paper were in no way theoretical but were based on the speaker's experience.

Mr. W. M. Butler, Yeatman High School, St. Louis, illustrated, by means of an opaque projector, how a 110 volt direct current, instead of batteries, is used for pupils' laboratory work in his school. The idea is a very practical one where dynamo current is available, thus eliminating the troublesome battery.

Mr. S. A. Douglass, Central High School, St. Louis, next gave an interesting demonstration of a sensitive flame.

The concluding paper of the session was on "High School Chemistry, What and How Much," by Mr. Fredus N. Peters, Central High School,

Kansas City. The paper appeared in the last issue of SCHOOL SCIENCE AND MATHEMATICS.

Saturday afternoon, the sections were taken by special car to visit the United Street Railways' extensive storage battery plant; afterward also to the Postal Telegraph offices to inspect the Rowland System of Multiplex Telegraphy by means of which eight messages may be sent in each direction at the same time on one wire. C. H. S.

TO OUR SUBSCRIBERS.

In accordance with the general custom of papers and educational journals we have continued to send the magazine to subscribers until ordered to discontinue and arrearages paid.

A new ruling of the Postmaster General forbids this and does not allow monthly magazines to be so sent longer than four months unless specifically renewed. This order was issued late in December and went into effect January 1, 1908.

We had on our books quite a number of accounts overdue at least a year and to whom from one to four letters had been sent.

As our new business was taking all of our time we could not attend to the collection of these accounts and so turned them over to the Newspaper Collection Agency of Chicago, a reputable firm.

By so doing we did not in the slightest intend to class anyone as a "dead beat" or "slow pay," but we had either to cross off these accounts and lose the money due us or else ask an agency to collect for us. As a matter of business we did the latter as we need every dollar due us to meet our bills.

During the past month we have sent notices of expiration to all subscribers whose subscriptions have expired. If you are one of these do not neglect to renew at once.

The supply of the January issue is practically exhausted. We will pay 25 cents cash or credit on subscription for each copy returned in good condition.

A SCIENCE SYMPOSIUM.

It is proposed to conduct in this journal at an early date symposiums on How, What and When to teach Botany, Chemistry, Earth Science, Physics and Zoology, and also one to bring out the relations existing between these subjects. There are many strong teachers in the country with excellent views on these matters. It is our purpose to invite the acknowledged leaders in the subjects mentioned to contribute articles of not exceeding 1,000 words expressing their ideas concerning the suggestions. Collecting and revising these papers we will be able to present to our readers the very best thought on the questions. Zoology will be presented first.

ERRATUM.

Page 93, line 11: For "if he solves them," read "if he proposes algebraic problems, and still more if he solves them."